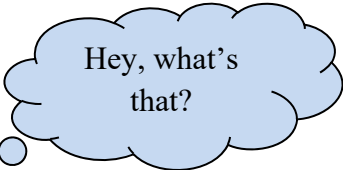


College algebra  
Class notes  
Matrices: Determinants (section 8.3)



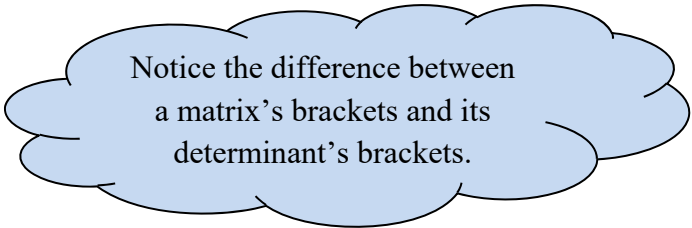
**Determinants** can be used to solve systems of equations by using something called **Cramer's Rule** (sometimes, but *not* always). There are other uses like for certain integration problems in calculus and certain geometric calculations but we won't go there.

The process of finding a determinant for a matrix gives us a single value. (It is actually a function since you get only one determinant for a given matrix.) We will play around with this idea to get our toes wet but not go much further. There is a fairly simple calculation, at least for 2x2 matrices. After that, it gets ugly... quite ugly... and we will use the calculator.

**Formula for the Determinant of a 2x2 Matrix:**

Define a square matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ . Here,  $a$ ,  $b$ ,  $c$ , and  $d$  are real numbers. The determinant of this

matrix is calculated as  $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ .



expl 1: Find the determinant below.

$$\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix}$$

expl 2: Find the determinant below.

$$\begin{vmatrix} 7 & 14 \\ 2 & 4 \end{vmatrix}$$

### **Finding Determinants on the Calculator:**

You must enter a square matrix (same number of rows and columns). Otherwise, it will give you an error.

1. Enter the **MATRIX** menu. It is the 2<sup>nd</sup> function of the  $x^{-1}$  button. Arrow over to **EDIT**.
2. Select **[A]** and press **ENTER**. You will then enter the order of the matrix, rows x columns.
3. Fill in the matrix with its entries.
4. Quit out to the home screen and re-enter the **MATRIX** menu. This time arrow over to **MATH**. Select **1:det(**. This is at the top of the list.
5. Re-enter the **MATRIX** menu and select **[A]** from the **NAMES** list.
6. Your home screen should now read **det([A])**. Press **ENTER** and it will output the matrix's determinant.

### **Formula for Determinant of 3x3 Matrix:**

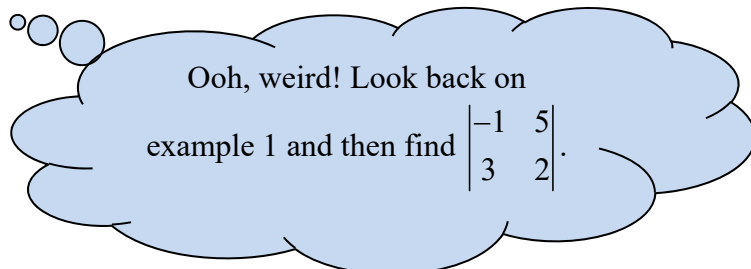
Nah, I'm joking. We aren't doing that by hand. Do those with the calculator.

expl 3: Find the determinant below.

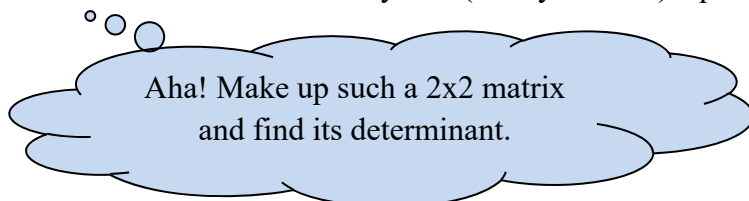
$$\begin{vmatrix} 2 & 3 & -1 \\ 5 & 4 & -6 \\ 8 & 2 & -3 \end{vmatrix}$$

## Properties of Determinants (or, What Some Smart People Learned Long Ago About Determinants):

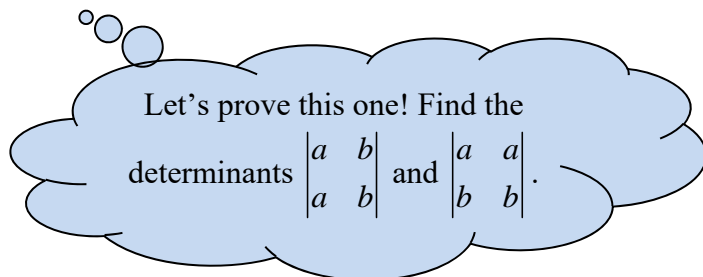
1. If any two rows (or any two columns) of a matrix are interchanged, then the value of the determinant changes sign.



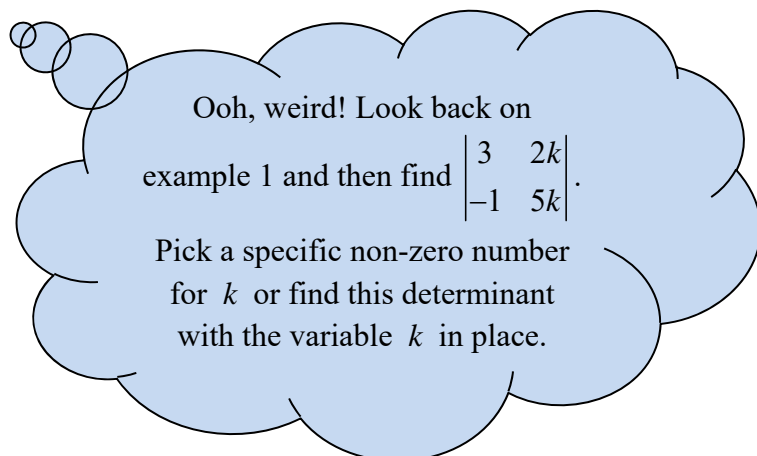
2. If all the entries in any row (or any column) equal 0, the value of the determinant is 0.



3. If any two rows (or any two columns) of a determinant have corresponding entries that are equal, the value of the determinant is 0.

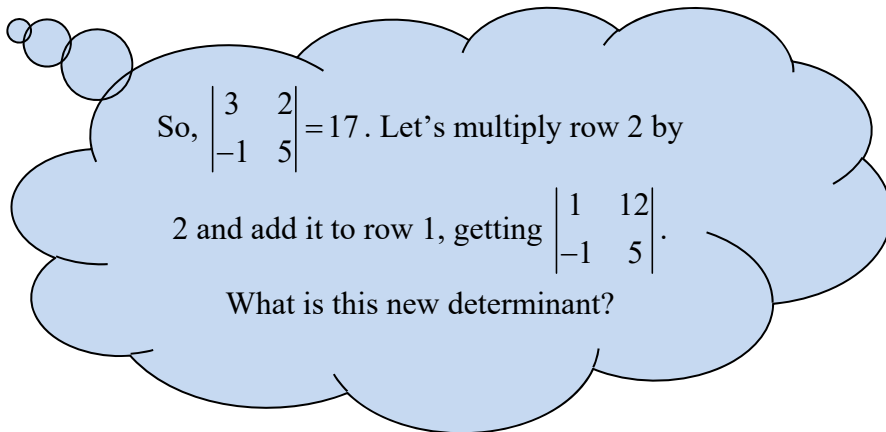


4. If any row (or any column) of a determinant is multiplied by a non-zero number  $k$ , then the value of the determinant is also changed by a factor of  $k$ .



**Determinant Property Number 5 (so juicy it needed its own page):**

5. If the entries of any row (or any column) of a determinant are multiplied by a non-zero number  $k$  and the result is added to the corresponding entries of another row (or column), then the value of the determinant remains unchanged.



expl 4: It is known that  $\begin{vmatrix} x & y & z \\ u & v & w \\ 1 & 2 & 3 \end{vmatrix} = 4$ . Find the values of the following determinants using their

properties discussed above. Give a brief explanation; which property(s) are you using?

a.)  $\begin{vmatrix} u & v & w \\ x & y & z \\ 1 & 2 & 3 \end{vmatrix}$

b.)  $\begin{vmatrix} x & y & z \\ 2 & 4 & 6 \\ u & v & w \end{vmatrix}$