

College algebra

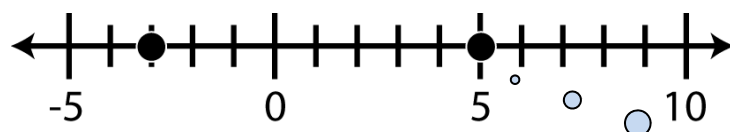
Class notes

Solving Absolute Value Equations and Inequalities (section 3.5)

Understanding what the absolute value of a number means will help us solve these problems.

Recall: Absolute value:

What does absolute value mean? For instance, when we say $|5| = 5$ or $|-3| = 3$, what are we saying about 5 and -3? Think about the real number line below.



What does absolute value tell us about a number?

Solving Absolute Value Equations:

So, if we write $|w| = 5$, what must w be? There are only two possibilities. What are they?

" w is the number that is 5 units from zero"

$w = ?$ or $w = ?$

Notice, we can change the equation $|w| = 5$ into two equations without absolute value involved, $w = -5$ or $w = 5$. We will use this idea to solve more complicated absolute value equations.

expl 1: Solve.

$$|2x - 11| = 5$$

This says " $2x - 11$ " is the number that is 5 units from zero. So what could this number " $2x - 11$ " be? Write your answers as two separate equations. Then solve the equations to find x .

Check your answers. Plug both your answers into the original equation.

$$|2x - 11| = 5 \quad \circ \quad \circ \quad \circ$$

Put 3 in to check it.

$$|2x - 11| = 5 \quad \circ \quad \circ \quad \circ$$

Put 8 in to check it.

expl 2: Solve.

$$|2x - 3| = -5$$

Think about this one before doing
any algebra to save work.

expl 3: Solve.

$$2|5x - 3| + 7 = 21$$

Isolate the absolute
value part before using
the procedure from
above.

Check your answers using the calculator. Make sure you always use the original equation to check answers.

$$2|5x - 3| + 7 = 21 \quad \cdot \quad \circ \quad \circ$$

Put 2 in to check it.

Absolute value is found in the **MATH** menu, under **NUM**.

$$2|5x - 3| + 7 = 21 \quad \cdot \quad \circ \quad \circ$$

Put $-4/5$ in to check it.

Calculator:

$$2 \text{ abs}(5(-4/5) - 3) + 7 \quad \boxed{\text{ENTER}}$$

MATH
Arrow over
to **NUM**

End the parentheses
that **abs(** started

Did you get 21?

Worksheet: Solving absolute value equations:

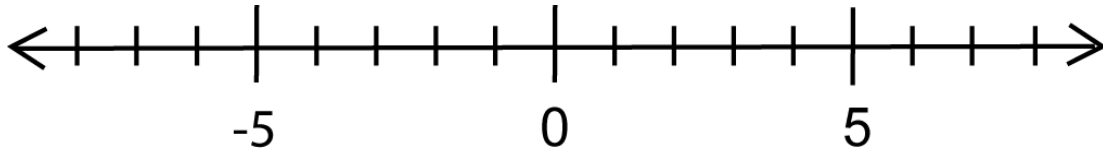
This worksheet explores why we solve these equations the way we do and gives us a bit of practice.

It is always a good idea to check your answers. Try to get into the habit on every problem.

Also, think about the general equation-solving process. To solve an equation, we rewrite it in simpler and simpler forms, until we get to a solution like $x = 5$. We saw that in solving quadratic equations by factoring and then breaking the factors into equations of their own, solving rational equations by eliminating the fractions, and here when we replace the original absolute value equation by two equations with no absolute value signs.

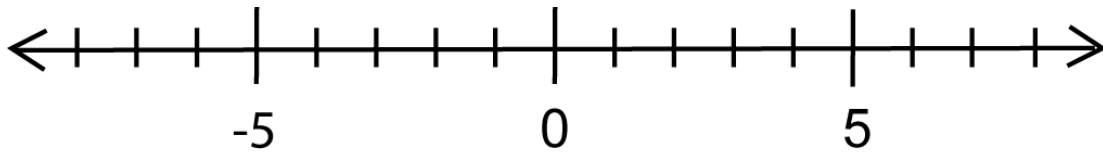
Solving Absolute Value Inequalities:

If a number's absolute value is more than 5, where does that place the number on the real number line? Let w be a number whose absolute value is more than 5. Graph where w could be on the number line below.



So if we have $|w| > 5$, how can we rewrite that inequality without absolute value signs?

If a number's absolute value is less than 5, where does that place the number on the real number line? Let w be a number whose absolute value is less than 5. Graph where w could be on the number line below.



So if we have $|w| < 5$, how can we rewrite that inequality without absolute value signs?

Let's use this general notion to solve some inequalities.

expl 4: Solve. Use interval notation and a real number line to denote the solution.

$$|3x + 5| > 10$$

expl 5: Solve. Use interval notation and a real number line to denote the solution.

$$\left| \frac{2x + 3}{2} \right| \leq 7$$