We have studied many types of functions and their graphs. Here is a new class of function.

College algebra
Class notes
Exponential Functions and Their Graphs (section 5.2)

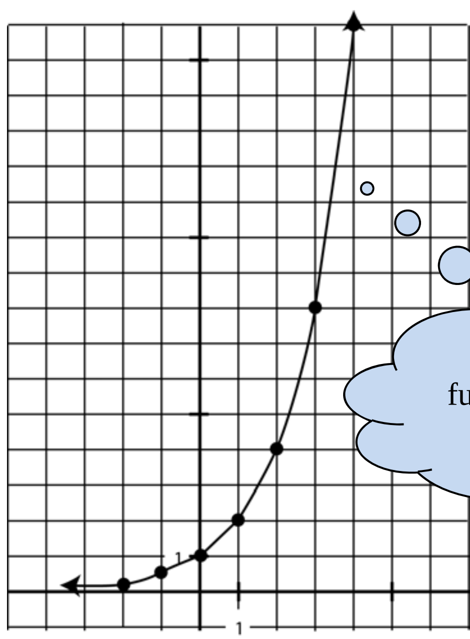
**Definition: Exponential Function:**

An **exponential function** is of the form $f(x) = a^x$ where $a$ is a positive real number not equal to 1. The number $a$ is called the **base**. Notice the variable $x$ is in the exponent position. This is what makes it an exponential function.

**Domain:** Since you could raise a positive number to any power and get a real number out, there are no real numbers we need to exclude from the domain. So **the domain of any exponential function is all real numbers**.

expl 1: Consider $y = 2^x$. Complete the table and then graph the function. [This is done for you.]

| $x$ | $y = 2^x$ |
|---|---|
| -2 | $2^{-2} = \frac{1}{4}$ |
| -1 | $2^{-1} = \frac{1}{2}$ |
| 0 | $2^0 = 1$ |
| 1 | $2^1 = 2$ |
| 2 | $2^2 = 4$ |
| 3 | $2^3 = 8$ |
| 4 | $2^4 = 16$ |

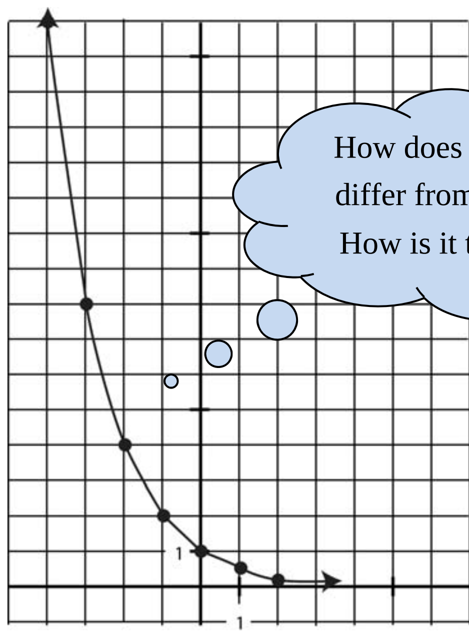



All exponential functions will have this general shape.

I call the shape "shloopy".

expl 2: Consider $y = \left(\frac{1}{2}\right)^x$. Complete the table and graph the function. [This is done for you.]

| ***x*** | $y = \left(\frac{1}{2}\right)^x$ |
|---|---|
| -4 | $\left(\frac{1}{2}\right)^{-4} = 2^4 = 16$ |
| -3 | $\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$ |
| -2 | $\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$ |
| -1 | $\left(\frac{1}{2}\right)^{-1} = 2^1 = 2$ |
| 0 | $\left(\frac{1}{2}\right)^{0} = 1$ |
| 1 | $\left(\frac{1}{2}\right)^{1} = \frac{1}{2}$ |
| 2 | $\left(\frac{1}{2}\right)^{2} = \frac{1}{4}$ |




How does this graph differ from $y = 2^x$? How is it the same?

What about the function makes the difference?

Why does $y = 2^x$ increase and $y = \left(\frac{1}{2}\right)^x$ decrease? What do you think makes the difference? Predict whether the following functions would increase or decrease.

a.) $y = 4^x$

b.) $y = 13^x$

c.) $y = \left(\frac{1}{3}\right)^x$

d.) $y = .67^x$

e.) $y = \left(\frac{5}{4}\right)^x$

**A closer look at the definition:**

An exponential function is of the form $f(x) = a^x$ where $a$ is a positive real number not equal to 1. Why does $a$ have to be positive and not 1?

If $a$ were negative, say -2, how would the graph look? If $a$ were 1, what would the graph look like? Find values for $f(x) = (-2)^x$ and $f(x) = 1^x$ to graph both functions.

These will **not** take on the general shape of an exponential function.

**Definition: Natural Exponential Function:**
This is a special exponential function whose base is $e$, an irrational number (meaning its decimal form does not terminate or repeat) approximately equal to 2.71828. The natural exponential function is $f(x) = e^x$.

expl 3: Find the following.

Calculator: You will see $e^x$ is the second function of the LN button on the left side.

a.) $e^{-5}$

b.) $\left(\frac{1}{e^3}\right)^2$

**Optional Worksheet: Working with Exponential Functions**
This worksheet will explore the two shapes we see with the graphs of exponential functions. It will also review simplifying exponents and using function notation. Solutions are available online.

**Review of Exponent Rules:** You will use many of these rules as you manipulate expressions.

Product rule: $a^m \cdot a^n = a^{m+n}$

Quotient rule: $\frac{a^m}{a^n} = a^{m-n}$

Power rule: $(a^m)^n = a^{m \cdot n}$

Power of a product rule: $(a \cdot b)^n = a^n \cdot b^n$

Power of a quotient rule: $\left(\frac{a}{c}\right)^n = \frac{a^n}{c^n}$

Zero exponent rule: $a^0 = 1$ (Here $a$ cannot be $0$ because $0^0$ is undefined.)

Negative exponent rule: $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ (if $a$ is non-zero and $n$ is an integer).

expl 4: Graph this function on the calculator. Use the standard window. This graph can be thought of as a transformation. Can you see the mother function $y = 3^x$ in this graph?

$y = 3 - 3^x$

We can use transformations to help graph these functions by hand. Recall the general shape of $f(x) = a^x$ when $0 < a < 1$ and when $a > 1$. Draw them right now below. What is the *y*-intercept of each graph? Do they have horizontal or vertical asymptotes?

expl 5: Use transformations to sketch the graph of the following functions. Check on your calculator.

a.) $y = 2^x + 1$

b.) $f(x) = -3^{x-2}$

**Worksheet: Exploring Exponential Functions:**

We will look at the graphs of exponential functions, including those involving transformations. We will also practice finding exponential values using the calculator.

expl 6: The value of a stock is given by the function $V(t) = 58\left(1 - e^{-1.1t}\right) + 20$ where $V$ is the value of the stock after time $t$, in months.

a.) Graph the function. Copy it here.

b.) Find $V(1)$, $V(2)$, $V(4)$, $V(6)$, and $V(12)$.

c.) After how long will the stock be worth $75?

Can you use the VALUE function under CALCULATE?

expl 7: Find the point(s) of intersection for the pair of functions below.

$$y = 2e^x - 3, \qquad y = \frac{e^x}{x}$$

expl 8: Use the compound interest formula to answer the following questions.
a.) Suppose \$150,000 is deposited in an account that pays 3.4% compound interest, compounded quarterly. Find the function for the amount $A(t)$ in the account after $t$ years.

$$A = P\left(1+\frac{r}{n}\right)^{n \cdot t}$$

$P$ = initial deposit
$A$ = amount in account after $t$ years
$r$ = interest rate (decimal form)
$n$ = number of compoundings per year
$t$ = number of years

b.) How much is in the account after 4 years?

c.) How many years does it take for the account to grow to \$200,000? Solve graphically.