College Algebra
Class Notes
Functions (section 1.2)

Function notation and the concept of function will follow you throughout algebra.

Idea behind Functions:

Equations like $y = 4x + 5$ or $x^2 + y^2 = 16$ show relationships between variables. These are called **relations**. They can also be represented by a table of values, a list of ordered pairs, or a graph which is just a picture of those ordered pairs. A function is a special kind of relation. Let's review some terminology to help us understand how they are special.

Definition: Domain: the set of all x values (that will give you a real number out for y)

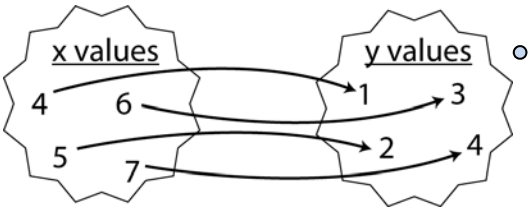
Definition: Range: the set of all y values (that you can get out for y)

x-values: inputs
y-values: outputs

But do you remember what a real number is? What would cause the output to be **non-real**?

expl 1: Consider the sets of ordered pairs and their illustrations below. Determine the domains and ranges of these relations.

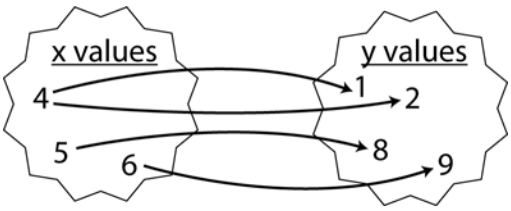
a.) (4, 1), (5, 2), (6, 3), and (7, 4)



Each x value is assigned to a **specific y value.**

What is the domain? What is the range? Write your answers in set notation.

b.) (4, 1), (4, 2), (5, 8), and (6, 9)



The x value of 4 is assigned to **two different y values**.

What is the domain? What is the range? Write your answers in set notation.

Definition: Function: a relation where every x value in the domain is assigned to **exactly one y value.**

In example 1 above, which relation is a function and which is not? Explain.

expl 2: Which of the following relations are functions?

Complete the table of values.

a.)

x	y
1	1
2	2
3	3
4	4

b.) (4, 5)
(5, 6)
(6, 7)
(7, 4)
(4, 7)

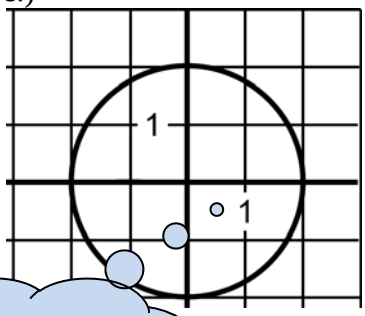
c.) $y = x^2$

x	y
-2	
0	
2	
4	

Solve for y.

d.) $x + 2y^2 = 1$

e.)



f.) *domain:* a set of family members
correspondence: their age
range: a set of integer values

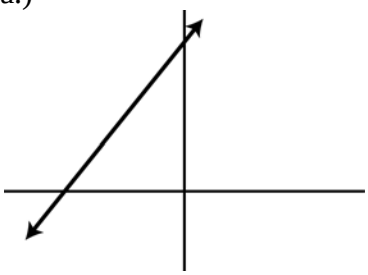
Highlight the points for the x value of 1. Does that help?

Vertical Line Test: Given a graph, the vertical line test will tell you if it is a function. If any vertical line could be drawn so that it crosses the graph more than once, then it is **not** a function. (The vertical line represents a single x value. If this vertical line hits the graph more than once, that x value has more than one y value and so the relation is not a function.)

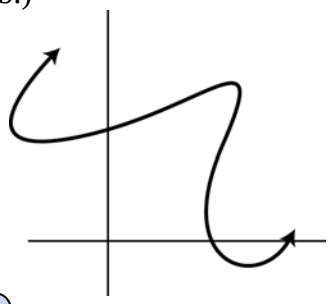
Try the vertical line test on part *e* above.

expl 3: Use the vertical line test to determine if the following are functions.

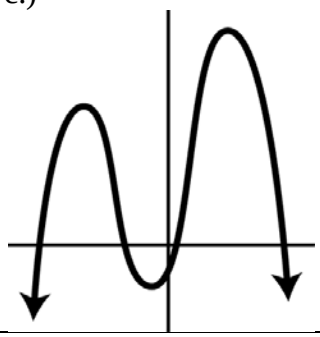
a.)



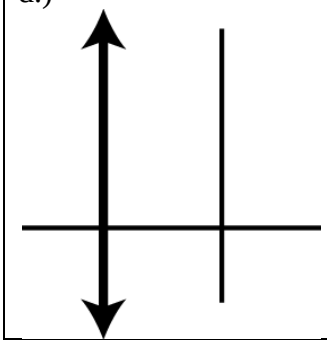
b.)



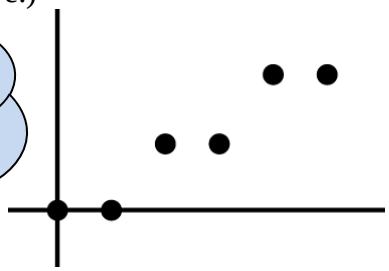
c.)



d.)



e.)



Draw multiple vertical lines to see if any x value has more than one y value.

Interpretation: You can think of a function in a few different ways.

1. a **relationship** between two variables, x and y,
2. a **rule** that tells you what to do to an x value to get out a y value, or
3. a **machine** that produces a y value when you input an x value.

In certain applications, one understanding of function may serve us better than the others.

$y = 2x^2 + 4$

Relationship: Every y value is equal to the x value squared, multiplied by 2, and added to 4.

Rule: This $y = 2x^2 + 4$ tells us what to do to an x value to make a y value. Can you picture a **machine** doing these operations?

Function notation:

Check to see if the following relationships are functions.

Does any x value result in more than one y value?

$y = 2x^2 + 4$		$y^2 = x$ (solved as $y = \pm\sqrt{x}$)	
x	$y = 2x^2 + 4$	x	$y = \pm\sqrt{x}$
-3		9	
0		16	
3		25	
Is y a function of x?		Is y a function of x?	

Since the first relationship is a function, we can use function notation to make sure everyone knows. So we replace the y with $f(x)$ to write $f(x) = 2x^2 + 4$. Sometimes we use different letters like $g(x)$ or $h(x)$, especially if we have more than one function.

We say y is a "function of x".

pronounced "f of x"

expl 4a: Find $f(0)$, $f(-2)$, and $f(5)$ for the function $f(x) = 2x^2 + 4$.

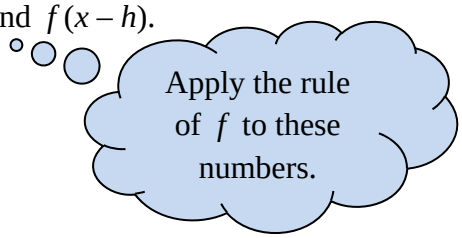
What we want are the y values when x is 0, -2, and 5.

How do you square -2 on the calculator?

Common Mistakes with Notation: As we use function notation in more complicated ways, understanding the notation and using it correctly will be crucial. For instance, in the previous example, we must never write $f(x) = 54$ or $f(5) = 2x^2 + 4$. Whatever you write in the parentheses should be substituted for x in the formula at the same time, on the same line.

expl 4b: Recall that the numbers 0, -2, and 5 are x values and the $f(x)$ outputs are their corresponding y values. Write your results from part a in ordered pair notation.

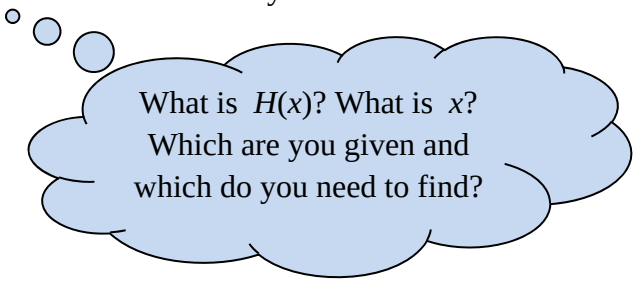
expl 4c: Consider our function $f(x) = 2x^2 + 4$. Find $f(-x)$, $f(x + 3)$ and $f(x - h)$.



Optional Worksheet: Investigating functions:
This worksheet practices determining if a relationship is a function and using function notation. Solutions are available on www.stlmath.com.

expl 5: Forensic science uses the function $H(x) = 2.59x + 47.24$ to estimate the height $H(x)$ of a woman (in centimeters) given the length x (in centimeters) of her femur bone.

a.) Estimate the height of a woman whose femur bone measured 40 cm. Round your answer to two decimal places.



b.) I am 5' 5" (or 165.1 centimeters). How long would you expect my femur to be? Round your answer to two decimal places.

Review of Interval Notation:

Do you remember interval notation? Provide each real number line graph and interval notation for these sets of numbers. The real number line graphs help me to visualize the sets.

"the numbers in between 0 and 4, not including 0, but including 4"

< is less than
> is greater than

Inequality Notation	Graph on Number Line	Interval Notation
$0 < x \leq 4$		
$x < -4$		
$x \geq 0$		
$x > 5$		
$4 \leq x$		

Sometimes the variable is on the right. How is that different?

smallest number in set , largest number in set

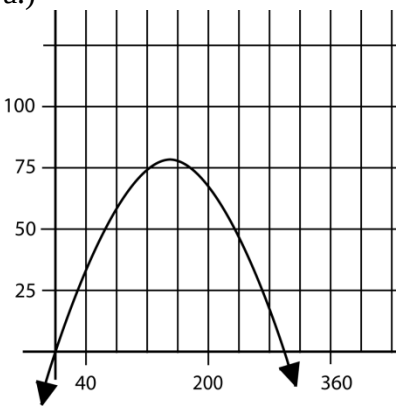
square bracket: includes endpoint
parenthesis: does not include endpoint

Which is x values?
Which is y?

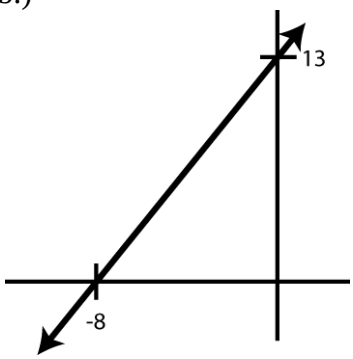
Finding domain and range:

expl 6: Find the domains and ranges for the various functions. Use interval notation or set notation where appropriate.

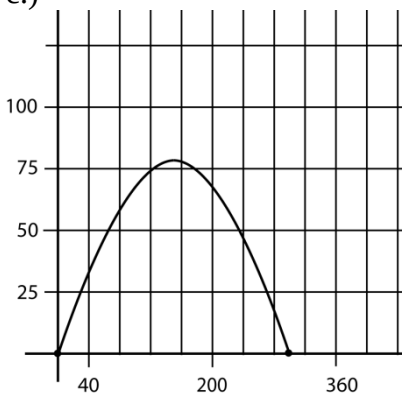
a.)



b.)



c.)



expl 7: Find the domains of the functions below. Use interval notation or set notation where appropriate.

a.) $y = \dfrac{3}{x+4}$

b.) $h(x) = \sqrt{2x+6}$

c.) $y = 5x + 9$

Instead of asking what x could be,
ask yourself what x **cannot** be.
The domain is everything else.

Optional Worksheet: Investigating functions 2:
This worksheet works you through function notation as well as domain and range. Solutions are available on www.stlmath.com.

This concept will follow you through algebra and further.

Factors versus terms:

terms: things we are adding (or subtracting)
expls: $\underline{x} + \underline{4}$ or $\underline{2x} + \underline{3}$ or $\underline{4x^2} + \underline{3x} - \underline{6}$

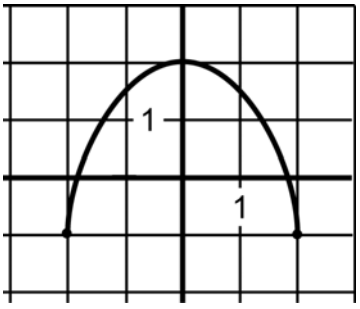
factors: things we are multiplying
expls: $\underline{5} \cdot \underline{x}$ or $\underline{3}(\underline{x+2})$ or $\underline{4} \cdot \underline{x^2}$

Could be thought of as $4 \cdot x \cdot x$ or $2 \cdot 2 \cdot x \cdot x$. What are the factors then?

The reason this distinction is important is because we will use these words a lot. When we simplify expressions, what we do and why (the rules that govern real numbers) depends a lot on if we are adding (or subtracting) versus if we are multiplying (or dividing).

Can you make up your own example of terms and factors?

expl 8: Use the graph of the function $f(x)$ to the right. Find the following values. Estimate if needed.



a.) $f(-1)$

b.) $f(0)$

c.) $f(2)$

Worksheet: Investigating functions 3:
We work on the definition of a function, domain, and finding function values graphically and algebraically.