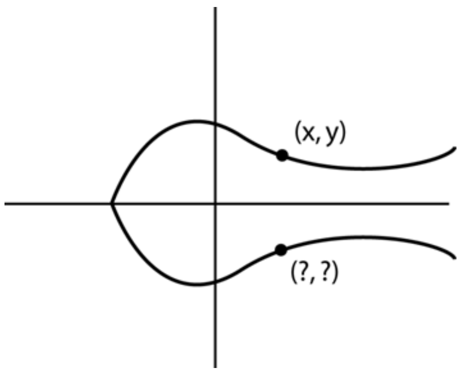
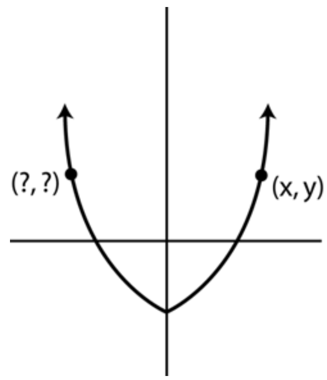
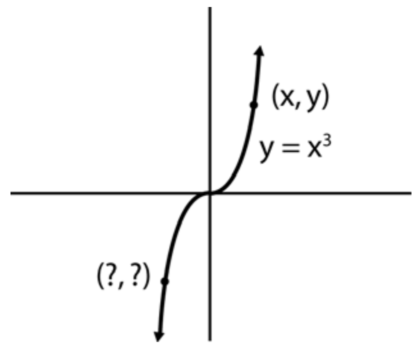
We will look at this graphically and also see how to algebraically prove symmetry.

College algebra
Symmetry and Even or Odd Functions
Section 2.4

Symmetry: Notice the symmetric nature of the following graphs. Which one is a mirror image of itself across the y-axis? the x-axis? Which one could be rotated 180 degrees about the origin and it would lie upon itself?







Can you figure out the coordinates of the unknown points? Write your answers above.

Algebraic tests for symmetry:

The pictures above help justify the following tests.

To test a relationship for symmetry about the …

***x*-axis:** Replace y with $-y$. If the equation is equivalent, then the relationship is symmetric with respect to the x-axis.

***y*-axis:** Replace x with $-x$. If the equation is equivalent, then the relationship is symmetric with respect to the y-axis.

origin: Replace x with $-x$ and replace y with $-y$. If the equation is equivalent, then the relationship is symmetric with respect to the origin.

expl 1: Graph the equation and determine visually if it is symmetric with respect to the x-axis, y-axis, and/or origin.

a.) $y = |3x| + 4$

b.) $y = \frac{2}{x}$

c.) $x^2 + y^2 = 25$

expl 2: Graph the equation and determine visually if it is symmetric with respect to the x-axis, y-axis, and/or origin. Then verify your conclusion algebraically.

$y^3 = 2x^2$

If you show symmetry about both the x-axis and y-axis, then the relation has symmetry about the origin too.

expl 3: Consider the following points and assume the graph has the given symmetry. Give another point that must also be on the graph.

a.) (5, -3); symmetric to y-axis

Plot each point.

b.) (4, 2); symmetric to the origin

c.) (4, 2); symmetric to the x-axis

Picture
$y = x^2$ and $y = x^3$.
Draw them here.

Definition: Even and Odd Functions:

If a function is symmetric about the y-axis, we call it **even**.

If a function is symmetric about the origin, we call it **odd**.

We can frame the earlier algebraic test in terms of function notation.

A function is even, if and only if for every x in the domain, we know $f(-x) = f(x)$.

A function is odd, if and only if for every x in the domain, we know $f(-x) = -f(x)$.

We'll use this to check whether a function is even, odd, or neither.

What does "if and only if" mean?

Could a function be both even and odd?

Why do they not have a word for a function that is symmetric about the x-axis?

expl 4: For the following functions, test if it is even, odd, or neither.

a.) $f(x) = x^3 + x$

Find $f(-x)$. Is it equal to $f(x)$ or $-f(x)$ or neither?

b.) $f(x) = 3x^2 - 5x^4$

c.) $f(x) = x^2 + 3x - 4$

Here's an optional problem to stretch your mind.

expl 5: Prove that the product of an odd function and an even function will always be odd.

Let $f(x)$ be an even function and $g(x)$ be an odd function.

We need to show

$$(f \cdot g)(-x) = -(f \cdot g)(x)$$

OR

$$f(-x) \cdot g(-x) = -f(x) \cdot g(x)$$