

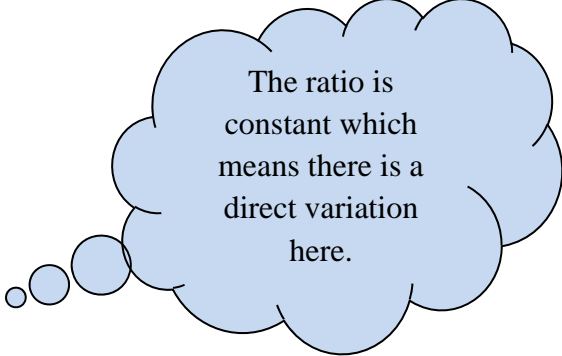
If two variables are related to each other by a constant ratio or product, we say we have a direct or inverse variation.

### Direct Variation

We will start with an example.

Margie gets paid \$10 per hour when she babysits. Fill in the table for the various number of hours given.

$x$ , number of hours	$y$ , total charge	Find $y/x$
1		
2		
3		
7		



The ratio is constant which means there is a direct variation here.

### Definition: Direct Variation:

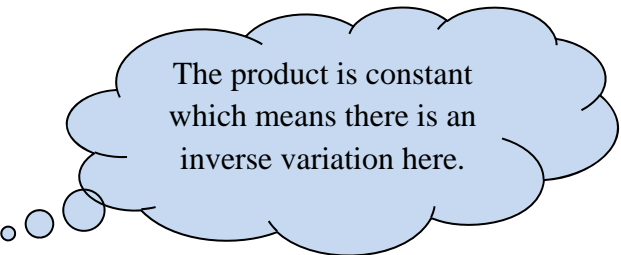
If a situation can be modeled by the linear function  $f(x) = kx$ , or  $y = kx$ , where  $k$  is a positive constant, we say that it is a **direct variation**. We could say  $y$  **varies directly as  $x$**  or  $y$  is **directly proportional to  $x$** . The number  $k$  is the **variation constant** or the constant of proportionality.

Can you think of any other variables that would be directly proportional?

## Inverse Variation

Again, let's look at an example. The area of a rectangle is 90 square meters. Fill in the table for the various widths given.

$x$ , width	$y$ , length	Find $x \cdot y$
5		
10		
15		
30		



The product is constant which means there is an inverse variation here.

### Definition: Inverse Variation:

If a situation can be modeled by the linear function  $f(x) = k/x$ , or  $y = k/x$ , where  $k$  is a positive constant, we say that it is an **inverse variation**. We could say  **$y$  varies inversely as  $x$**  or  **$y$  is inversely proportional to  $x$** . The number  $k$  is the variation constant or the constant of proportionality.

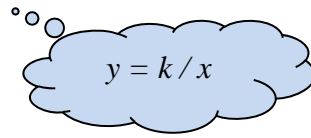
Can you think of any other variables that would be inversely proportional?

expl 1: Find the variation constant and the equation of variation for the given situation.

a.)  $y$  varies directly as  $x$ , and  $y = 54$  when  $x = 12$


$$y = kx$$

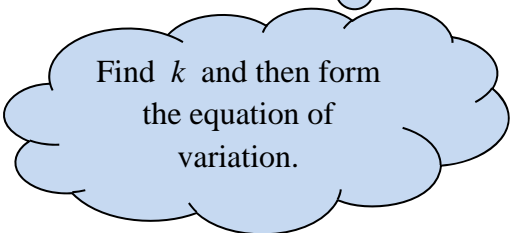
b.)  $y$  varies inversely as  $x$ , and  $y = 12$  when  $x = 5$


$$y = k/x$$

expl 2: The weight  $W$  that a horizontal beam can support varies inversely as the length  $L$  of the beam. Suppose an 8 meter beam can support 1200 kg. How many kilograms can a 14 meter beam support?



$y = kx$  OR  $y = k/x$  ?

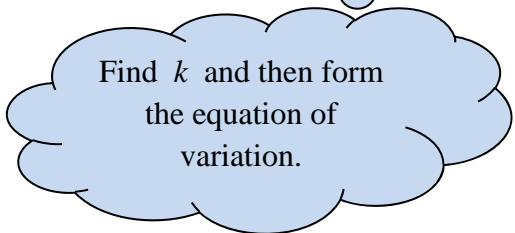


Find  $k$  and then form  
the equation of  
variation.

expl 3: The relative aperture, or f-stop, of a 23.5-mm diameter camera lens is directly proportional to the focal length  $F$  of the lens. If a focal length of 150 mm has an f-stop of 6.3, find the f-stop of this lens with a focal length of 80 mm.



$y = kx$  OR  $y = k/x$  ?



Find  $k$  and then form  
the equation of  
variation.

## Combined Variation

There are three other types of variation we will study.

1.  **$y$  varies directly as the  $n$ th power of  $x$**  if there is some positive constant  $k$  such that

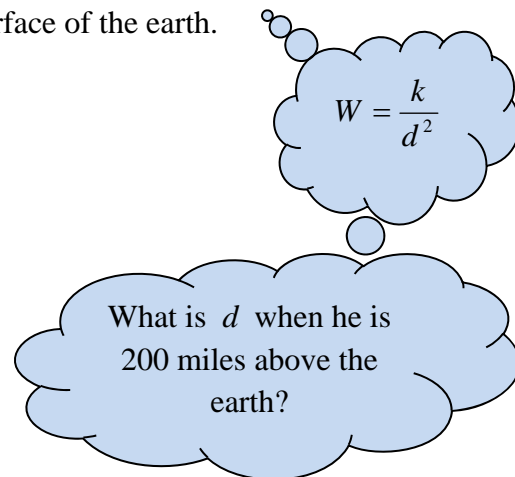
$$y = kx^n.$$

2.  **$y$  varies inversely as the  $n$ th power of  $x$**  if there is some positive constant  $k$  such that

$$y = \frac{k}{x^n}.$$

3.  **$y$  varies jointly as  $x$  and  $z$**  if there is some positive constant  $k$  such that  $y = kxz$ .

expl 4: The weight  $W$  of an object varies inversely as the square of the distance  $d$  from the center of the earth. At sea level (3978 miles from the center of the earth), an astronaut weighs 220 pounds. Find his weight when he is 200 miles above the surface of the earth.



expl 5: Find the variation constant and the equation of variation for the given situations.

- a.)  $y$  varies jointly as  $x$  and  $z$ , and  $y = 56$  when  $x = 7$  and  $z = 4$

- b.)  $y$  varies jointly as  $x$  and  $z$  and inversely as  $w$ , and  $y = 14$  when  $x = 3$ ,  $z = 2$ , and  $w = 2$