Solving a system of equations means to find the point that makes **all** of the equations true.

College algebra
Class notes
Solving Systems of Linear Equations in Two Variables (section 6.1)

To solve a system of equations means to find the one point that satisfies both equations. We will learn three methods, one graphical and two algebraic methods.

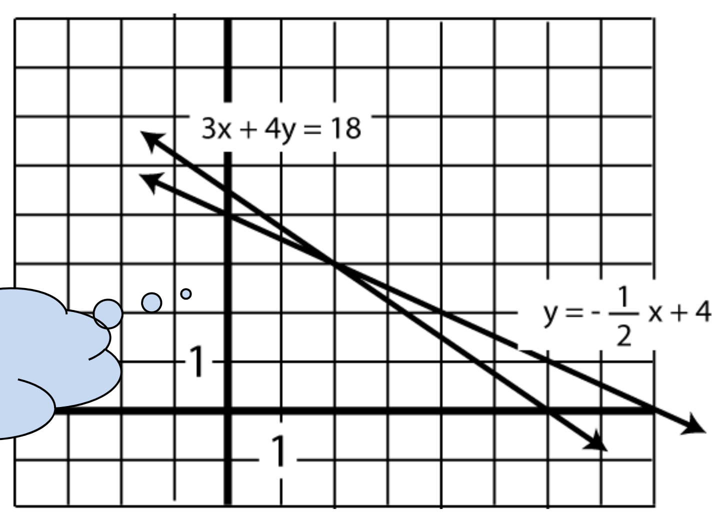
Graphically Solving Systems of Linear Equations:
This method uses the facts that …

the graph of a linear equation is always a straight line, and
all the points on the graph of a line satisfy the equation (that is, makes it true).

Now, if we want to find the point that satisfies two different equations, we would start by graphing both lines. Then what would we look for on the graph? See the example below.

expl 1: Solve the system. The two lines are graphed for you.

$$3x + 4y = 18$$
$$y = -\frac{1}{2}x + 4$$

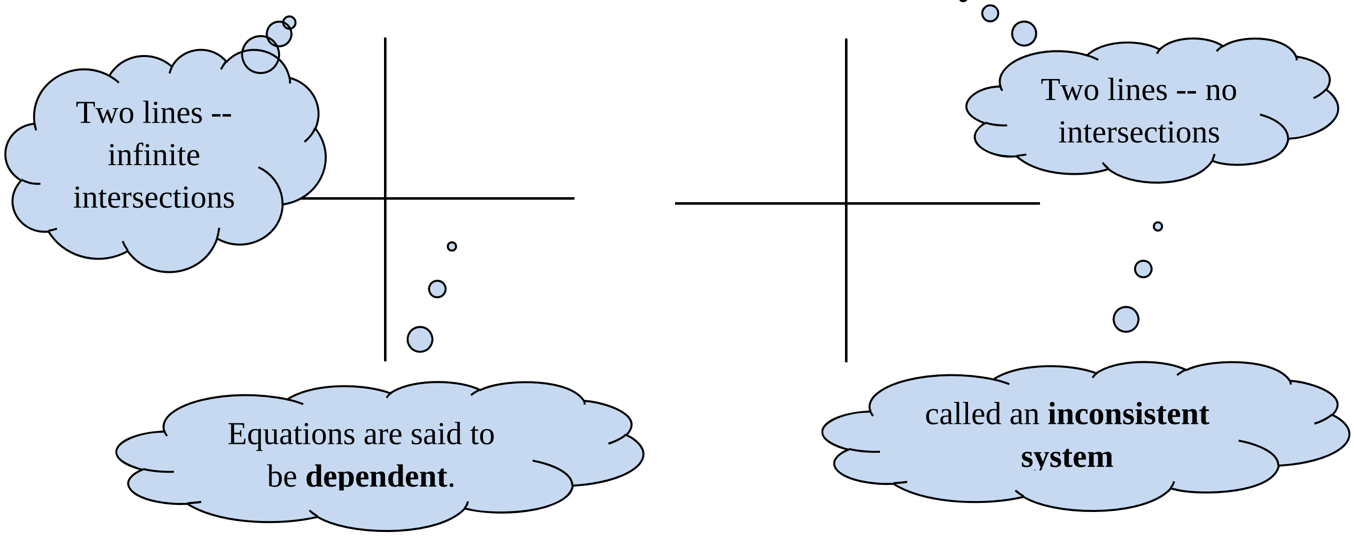


What is the solution to the system?

Try your solution in both equations. Does it make **both** equations true?

Systems with infinitely many or no solutions:
So, the solution to a system of equations is the intersection of the two lines. The preceding example has one solution (corresponding to one intersection). There are two other possibilities, an infinite number of solutions or no solutions. Think about how those systems would look and draw possible graphs below.

Infinite solutions   No solutions



Is it possible for two lines to intersect in exactly two or three points? Explain.

Optional Worksheet: Solving linear systems graphically:

This worksheet contains examples for this method including tips on how to graph the lines. It has examples of the three possibilities we face: one, none, and an infinite number of solutions.

Optional Worksheet: Solving systems of equations graphically: Calculator worksheet:

This worksheet shows solving equations for y in order to put them into the calculator, using the Intersect function on the calculator (TI-82, 83, 84, 85, and 86) to find the exact intersection, and finding an appropriate window for our graphs.

Algebraically Solving Systems of Linear Equations by Substitution:
Consider our dilemma. We are given two equations but we can’t solve either for x because that darn y is in the way. If only we had just one equation with just one variable. Then we could solve it like we are used to. Look at the example here.

expl 2: Solve the system.

$$3x + 4y = 18$$
$$y = -\frac{1}{2}x + 4$$

How do you turn these two equations with two variables into one equation with one variable?

Once you find x, how would you find the y that goes with it? Do it now. Write your solution as an ordered pair.

Algebraically Solving Systems of Linear Equations by Elimination (or Addition):
We will simply add the equations. This will eliminate one of the variables. We then solve for the variable that’s left. Remember to write your solution as an ordered pair.

expl 3: Solve the system by elimination.

$$3x + 2y = 2$$
$$5x - 2y = 14$$

Add the left sides, add the right sides, and set them equal.

If $a = 2$ and $b = 5$, then $a + b = 2 + 5$. Would you agree?

expl 4: Solve the system by elimination.

$x + 4y = 14$
$5x + 3y = 2$

Adding now will not eliminate a variable. How can we change one of the equations so that x will be eliminated when we add?

expl 5: Solve the system by elimination.

$9x - 3y = 12$
$12x - 4y = 18$

It's probably easiest to change both equations.

expl 6: Solve by the substitution method.

$$2y = x + 2$$
$$6x - 12y = 0$$

It might help to think of the second equation as $\mathbf{6x - 6 \cdot 2y = 0}$. Why?

What was the solution last time we got a result like this?

expl 7: Solve.

$$\frac{1}{4}x - 2y = 1$$
$$x - 8y = 4$$

What was the solution last time we got a result like this?

The system above has infinite solutions. We have to write the points that satisfy both equations in ordered pair notation just like before. Solve one of the equations for y and then write the solution as (x, ---). Similarly, we could solve one the equations for x and then write the solution as (---, y). Do it now.

expl 8: Solve.

$$-2.5x - 6.5y = 47$$
$$.5x - 4.5y = 37$$

Multiply by 10 first if you do not like decimals.

Applications:

expl 9: Turi's Tee Shirt Shack sold 36 shirts one day. Each short-sleeved shirt costs \$12 and each long-sleeved shirt costs \$18. Total receipts for the day totaled \$522. How many of each type of shirt did they sell?

Let x be the number of short-sleeved shirts and y be the number of long-sleeved shirts.

Form two equations, one from number of shirts and one from cost.

expl 10: Bob's boat travels 45 miles downstream in 3 hours. The return trip upstream (going the same route) takes 5 hours. Find the speed of the boat and the speed of the current.

Let x be the speed of the boat and y be the speed of the current.

$d = r \cdot t$

Form two equations, one from the downstream trip and one from the upstream trip.

Definition: Break-even Point:
The **break-even point** for a company is the point at which their cost equals their revenue (money they bring in). Algebraically, this is when $C = R$. Usually we find how many units they should produce and sell to break even.

expl 11: Find the break-even point for a company with the following cost C and revenue R (in dollars). Let x represent the number of units produced and sold.
$C = 15x + 12{,}000$
$R = 18x - 6{,}000$

Set up and solve $C = R$.