College algebra Class notes


Polynomial Functions and Regression (section 4.1)

## Definition: Polynomial function:

A polynomial function is a function that can be written in the form $f(x)=a_{n} x^{n}+a_{n-1} x^{n-1}+a_{n-2} x^{n-2}+\ldots+a_{1} x+a_{0}$ where $a_{n}, a_{n-1}, a_{n-2}, \ldots, a_{0}$ are real numbers and the exponents are whole numbers.


$$
\begin{array}{ll}
\text { expls: } & y=a x^{2}+b x+c \\
& f(x)=m x+b \\
& y=\frac{1}{2} x^{5}-4 x^{4}+2 x^{3}-7 \\
& f(x)=\sqrt{5} x^{3}+4 x^{2}-x+10 \\
& g(x)=\sqrt{2} x^{2}-5 x^{7}+4 x^{6}-\sqrt{12} \\
y=(2 x+3)(x-1)
\end{array}
$$

Some counterexamples follow. Can you tell why they are not polynomials? How do they not fit the definition?
counterexpls: $y=4 x^{-3}+2 x-7, \quad f(x)=\frac{1}{2} \sqrt{x}+4 x^{2}, \quad y=\frac{14 x^{2}+x}{x^{3}-8}$

## Terminology:

$a_{n}, a_{n-1}, a_{n-2}, \ldots, a_{0}$ are called coefficients
$a_{0}$ is called the constant coefficient
$a_{n}$ is called the leading coefficient

$a_{n} x^{n}$ is called the leading term
$n$ is called the degree
$x$ is called the variable

## Characteristics of Graphs of Polynomial Functions:

1.) The domain will always be "all real numbers". (We have no square roots or division by 0 in polynomial functions. So nothing stands in the way of an $x$ value "working" in the function.)
2.) The graph is continuous. This means you could trace the whole graph from the left end to the right end without lifting your pencil.
3.) The graph has no sharp corners. It is a smooth curve.
4.) The last major characteristic of a polynomial graph is its end behavior. End behavior answers the question, "what is happening to the $y$ values at the (left and right) ends of the graph?" We'll investigate end behavior next.


## Worksheet: Polynomial functions: End behavior:

This worksheet explores the end behavior of the graphs of polynomial functions. We look at what is happening to the $y$ values at the (left and right) ends of the graph. In other words, we are interested in what is happening to the $y$ values as we get really large $x$ values and as we get really small (negative) $x$ values.

Fill in this table as a summary of what you learned from the worksheet. Recall how the leading term alone will determine a polynomial function's end behavior. This is sometimes referred to as the Leading Term Test.

|  | Leading coefficient is negative | Leading coefficient is positive |
| :--- | :--- | :--- |
| Degree <br> is odd |  |  |
|  |  |  |
| Degree <br> is even |  |  |
|  |  |  |

You will see these symbols as shorthand for the four types of end behavior. Which one corresponds to which end behavior above?

expl 1: Determine the leading term, leading coefficient, and the degree of the polynomial. Then classify it as linear, quadratic, cubic, or quartic.
a.) $r(x)=-5 x^{4}+2 x^{3}-7$

b.) $f(x)=3-5 x^{2}+9 x^{3}$
expl 2: Find the end behavior of each function. Write the end behavior using the notation shown in the worksheet "Polynomial functions: End behavior" and then select the picture below that describes the end behavior.
a.)


d.)

a.) $r(x)=-5 x^{4}+2 x^{3}-7$
b.) $f(x)=3-5 x^{2}+9 x^{3}$

## Worksheet: Polynomial functions: End behavior 2:

This worksheet will help you practice use the procedure and notation described here.

## Zeros of Polynomial Functions:

Recall: Definitions: Zero and $x$-intercept:
An $x$-intercept is where the graph hits the $x$-axis. Since the $y$ value is 0 at these points, these are also the $x$ values that make $f(x)=0$. A number that makes $f(x)=0$ is called a root or zero.
expl 3: Use substitution to determine if the values 2 , 3 , and -1 are zeros of the function $g(x)=x^{4}-6 x^{3}+8 x^{2}+6 x-9$.

expl 4: Consider the function $f(x)=x(x-3)^{2}(x+1)$. Algebraically solve $f(x)=0$ to find the zeros of this function.

What did you get? Notice how these zeros are tied directly to the factors of $f(x)$. The following theorem spells this relationship out.

## Zero / Factor Theorem:

Let $f$ be a polynomial function and $r$ be a real number in its domain. The expression $x-r$ is a factor of $f$ if and only if $r$ is a zero of $f$.


Definition: Multiplicity of a zero: The multiplicity of a zero (or root) is the number of times its corresponding factor appears in the factored form of the polynomial.
expl 5: In the last example, we found the zeros of this function. State each zero's multiplicity. $f(x)=x(x-3)^{2}(x+1)$

It turns out that the multiplicity of a zero impacts how the graph appears at that $x$-intercept. We will not be exploring this but the book looks into it a bit.
expl 6: Find the zeros of the function and state their multiplicities. $y=3 x^{3}+x^{2}-48 x-16$

expl 7: Use a calculator to find the zeros of the function. Approximate the values to three decimal places. Include a quick graph of the function with the zeros labeled.
$f(x)=2 x^{3}-x^{2}-14 x-10$
expl 8: A dog's life span is typically much shorter than that of a human. The cubic function $d(x)=0.010255 x^{3}-0.340119 x^{2}+7.397499 x+6.618361$ where $x$ is the dog's age, in years, approximates the equivalent human age in years. Estimate the equivalent human age for a dog that is 12 years old.

## Quadratic and Higher Degree Regression Equations:

We studied regression before. We saw how the pattern of a scatter plot of points could be represented by a single linear equation. But not all scatter plots show a linear pattern. Look at the plots below. Draw in a curve that mimics the pattern of points.


Definition: Coefficient of Determination, $\mathbf{R}^{\mathbf{2}}$ :
The coefficient of determination is used similarly to the correlation coefficient seen with linear regression. When we try to fit various types of regression (quadratic, cubic, or quartic) to a set of data, the coefficient of determination will tell us which gives us the best fit. The regression equation that gives us an $R^{2}$ value closest to 1 will be the one we choose to use.
expl 9: The number of foreign adoptions in the U.S. has declined in recent years, as shown in the table to the right.
a.) Use your calculator to fit quadratic, cubic, and quartic functions to this data. Let $x$ represent the number of years since 2000. Round your equations' coefficients to three decimal places. Use the $\mathrm{R}^{2}$ values to determine which function is the best fit.
b.) Use the function from part $a$ to estimate the number of U.S. foreign adoptions in 2010.

| Year, $\boldsymbol{x}$ | Number of U.S. Foreign <br> Adoptions from Top 15 <br> Countries, $\boldsymbol{y}$ |
| :---: | :---: |
| 2000,0 | 18,120 |
| 2001,1 | 19,087 |
| 2002,2 | 20,100 |
| 2003,3 | 21,320 |
| 2004,4 | 22,911 |
| 2005,5 | 22,710 |
| 2006,6 | 20,705 |
| 2007,7 | 19,741 |
| 2008,8 | 17,229 |
| 2009,9 | 12,782 |

expl 10: Use your regression equation to estimate the number of adoptions in the year 2050. Why does this value not make sense?

## Worksheet: Quadratic (and higher order) Regression on Your Calculator (TI-82, 83, or 84):

This worksheet provides an example and step-by-step instructions for finding a quadratic, cubic, and quartic regression equations on your calculator. The higher degree regression equations are done similarly. On these problems, you will want to compare higher orders of regression to find which fits the data best. This is mentioned on the worksheet.

