We will add to our knowledge of what the graphs look like and where the real zeros might exist.

College algebra
Class notes
Graphing Polynomial Functions and the Intermediate Value Theorem (section 4.2)

We have learned a lot about a function's real zeros which are the graph's x-intercepts. We also have studied the end behavior of polynomial graphs. What else might we need to know to accurately graph these functions? Here are some main facts that we will use.

Maximum number of zeros and turning points:

Let $P(x)$ be a polynomial function of degree n. The graph of this function has

1.) at most n real zeros, and thus at most n x-intercepts, and
2.) at most $n-1$ turning points.

A turning point is another name for a local (or relative) maximum or minimum.

expl 1: Determine the maximum number of real zeros (and therefore the maximum number of x-intercepts) and the maximum number of turning points the functions below can have.

a.) $f(x) = 4x^6 + 7x^3 - 5x^2 + 3x - 8$

b.) $h(x) = -3.5x^3 + 4.3x^2 - 3.2x + 3.6$

expl 2: Use your knowledge of the end behavior of polynomial functions and y-intercepts to match the following functions to their graphs.

A.) $f(x) = x^2 - 4x - 1$

B.) $p(x) = -2x^4 + 3x^2 - 5$

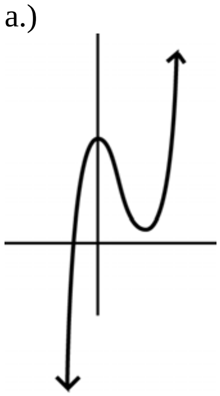
C.) $f(x) = -4x^3 + 2x^2 + 3$

D.) $r(x) = -2x^6 + 5x^3 - 4x + 8$

E.) $g(x) = x^3 - 3x^2 + 5$

F.) $y = -3x^5 - 5x^2 + 3x - 4$

a.)



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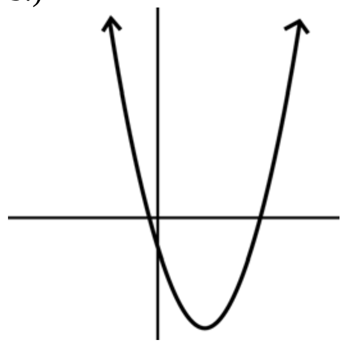

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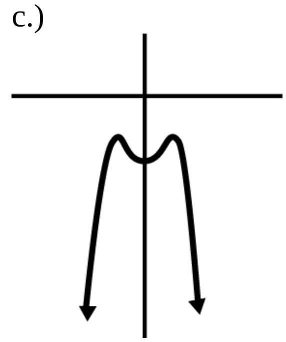

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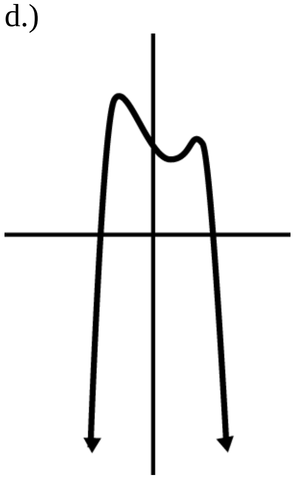

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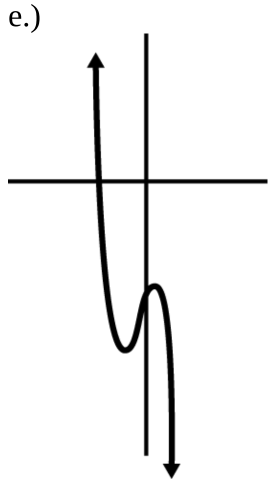
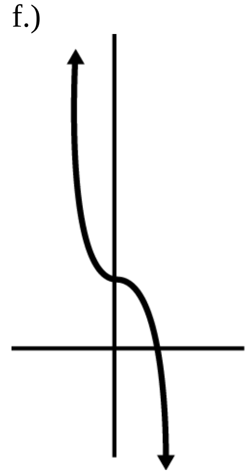
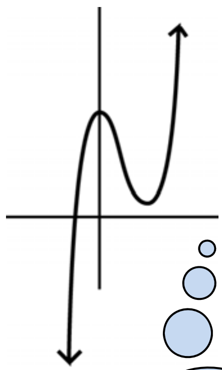
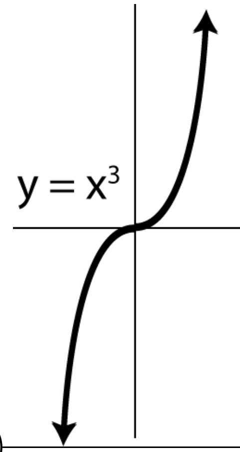
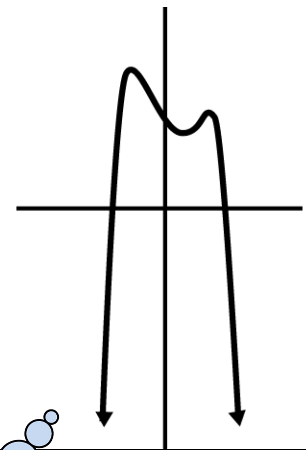


f.)



expl 3: Mark the turning points, if they exist, on the graphs below.

a.)	b.)	c.)
		

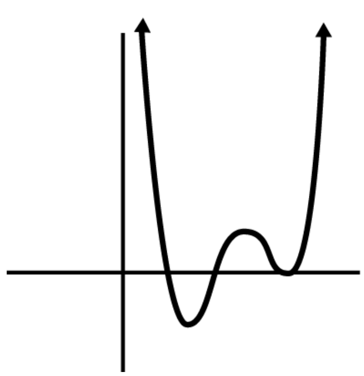
A turning point is also where a graph changes from increasing to decreasing or vice versa.

Recall that there can be no more turning points than one less than the degree of the polynomial.

Verifying a Complete Graph:
Knowing what we know (the end behavior, the maximum number of zeros, and the maximum number of turning points that a function can have) helps us graph it by hand or determine if we have a complete graph on the calculator. For instance, let's say we graph $y = x^4 - 11x^3 + 42x^2 - 64x + 32$. The graph is shown below. Using what you learned from this section, explain how you know it has to be a complete graph.

A complete graph shows all intercepts, turning points, and the end behavior.

$y = x^4 - 11x^3 + 42x^2 - 64x + 32$



Graphing a polynomial function by hand:

1. Use the leading term to determine the end behavior.
2. Find the zeros by solving $f(x) = 0$. Any real zeros are the x-intercepts of the graph.
3. Use the x-intercepts to break the domain into intervals. Choose an x value in each interval and find its y value. This will help us see the general shape of the graph.
4. Find the y-intercept by finding $f(0)$.
5. Sketch the graph connecting the intercepts and various other points.
6. Use the facts of this section to give a quick check to your graph. Your graph should have no more than n zeros and $n - 1$ turning points. You also want to check the multiplicity of your zeros and verify that the graph touches or crosses the x-axis appropriately.

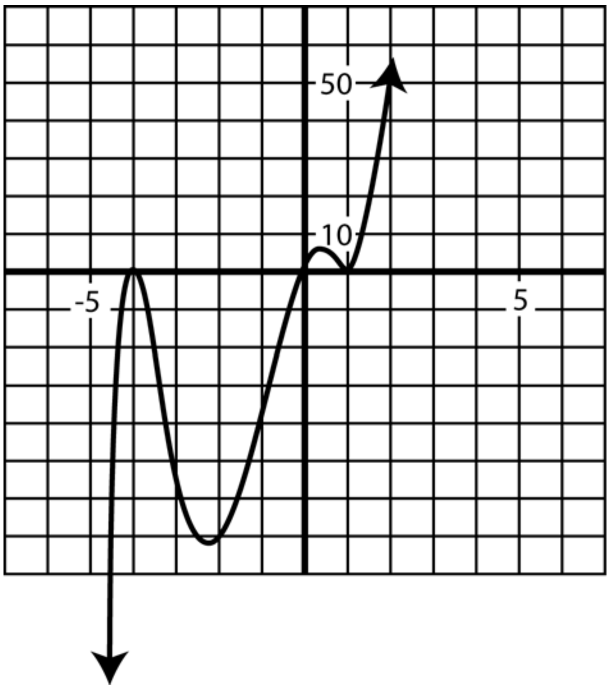
expl 4: Graph the polynomial function. Follow the steps outlined above.

$y = x(x-1)(x+3)(x+5)$

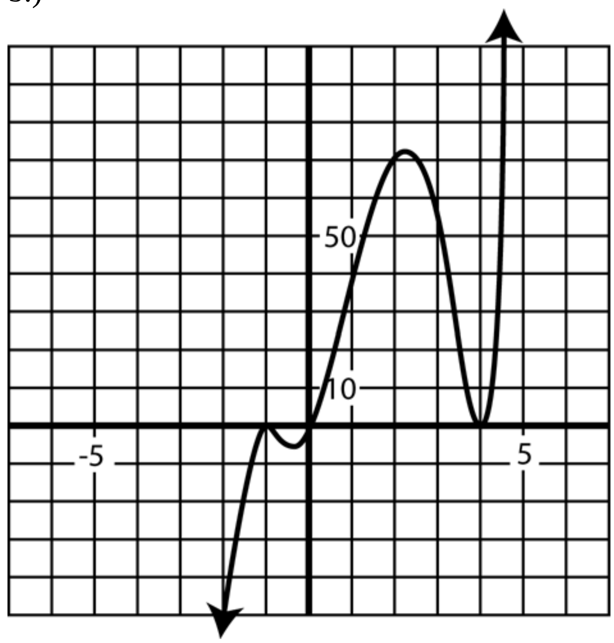
expl 5: Select the graph of the polynomial function below.

$f(x) = -x(x-1)^2(x+4)^2$

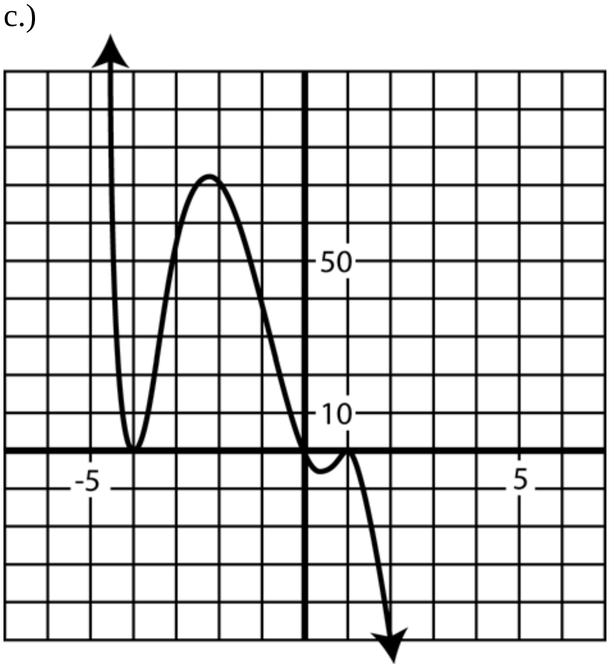
a.)



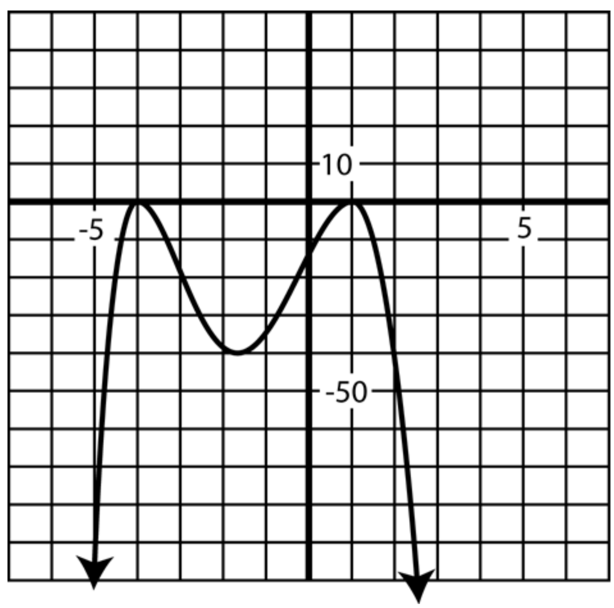
b.)



c.)



d.)



expl 6: Graph the piecewise function.

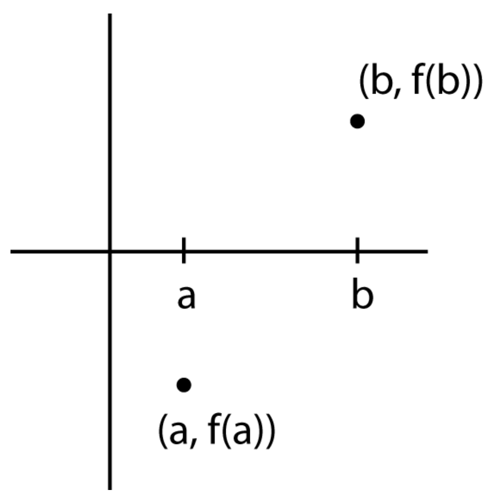
$$h(x) = \begin{cases} -x^2, & x < -2 \\ x+1, & -2 \le x < 0 \\ x^3 - 1, & x \ge 0 \end{cases}$$

Visualize $y = -x^2$ on its entire domain. But graph it only where $x < -2$.

The Intermediate Value Theorem:

For any polynomial function *P(x)* with real coefficients, consider two x values a and b such that $a \neq b$. If *P(a)* and *P(b)* are of opposite signs, then the function has a real zero between a and b.

Challenge yourself to disprove this theorem. Here are two values a and b as described above. Can you draw in a polynomial function that contains both but not a zero in between?



expl 7: Use the Intermediate Value Theorem to determine, if possible, whether the function has a real zero between a and b.

a.) $f(x) = x^3 + 3x^2 - 9x - 13$; $a = -5$; $b = -4$

If the signs are the same, you learn nothing. You **cannot** assume there is not a zero. Why not?

b.) $f(x) = x^3 + 3x^2 - 9x - 13$; $a = 1$; $b = 2$

expl 8: If the signs of $f(a)$ and $f(b)$ are **not** opposite, you cannot conclude anything. To illustrate this, draw a picture of a polynomial function where the following occurs.

a.) $f(a)$ and $f(b)$ are of the same sign **and there is no real zero** between a and b.

b.) $f(a)$ and $f(b)$ are of the same sign **and there is a real zero** between a and b.