College algebra Class notes


Graphs of Quadratic Functions (section 3.3)
Definition: Quadratic Function: A quadratic function is a function that could be written in the form $f(x)=a x^{2}+b x+c$ where $a$ is not zero. We will also use the form $f(x)=a(x-h)^{2}+k$ where $a$ is not zero. (Completing the square will get us from the first form to the other.)

We will investigate the graphs of quadratic functions such as $y=(x+5)^{2}, f(x)=4 x^{2}$, and $g(x)=-3(x-4)^{2}+9$. These can be considered transformations on the basic function $y=x^{2}$. So let's start there...

Graph $y=x^{2}$ from memory. Recall the nice symmetric shape. Also, notice how its vertex is exactly on the origin.


Definition: Vertex and Axis of Symmetry: The vertex of a parabola is where it bends. We will use ordered pair notation to write the vertex. The axis of symmetry is the vertical line that goes through the vertex. Notice the graph is symmetric about that imaginary line.

When a quadratic function is in the form $f(x)=a(x-h)^{2}+k$, you can determine its graph by thinking of the various transformations that you apply to $y=x^{2}$.
expl 1: Use transformations to describe how the graph of $f(x)=2(x-5)^{2}+3$ differs from $y=x^{2}$. From that information, determine the vertex and axis of symmetry of $f$. Also, draw a quick graph of $f$.

## Vertex, Axis of Symmetry, and Orientation of a Parabola:

In general, what would the vertex of the function $f(x)=a(x-h)^{2}+k$ be? What is the equation of the axis of symmetry? What about the formula determines if the parabola opens up or down? (This is called orientation. Think of a downward orientation as the result of a reflection about the $x$-axis.)


Alternative Formula for Vertex of a Parabola: It so happens that if a quadratic function is in the form $f(x)=a x^{2}+b x+c$, the $x$-value of the vertex can be found by calculating $x=\frac{-b}{2 a}$.


Recall: Finding $x$ - and $y$-intercepts: To find the $x$-intercept of a function, substitute 0 for $y$ and solve for $x$. To find the $y$-intercept, substitute 0 for $x$ and solve for $y$. (This is true for any type of function, not just quadratic ones.) Finding the $x$-intercept may require the quadratic formula. That is a good use of the calculator program since it gives decimal answers.

For the generic function $y=a x^{2}+b x+c$, what will the $y$-intercept always be?

## Graphing Quadratic Functions:

We will graph quadratic functions by hand by finding and plotting the vertex and $y$-intercept and using the orientation (and its symmetry) to fill in the rest of the parabola.

We will work with two different forms of a quadratic function, $f(x)=a(x-h)^{2}+k$ and $f(x)=a x^{2}+b x+c$.
expl 2: Find the vertex and axis of symmetry for this quadratic function. Determine if the vertex is a minimum or a maximum. Graph the function.

$$
g(x)=3(x+4)^{2}-5
$$

So, answer the following questions:
a.) What is the vertex of the parabola?
b.) What is the axis of symmetry?

c.) Is the vertex a maximum or minimum value? What is the extremum value?
d.) Graph the function.

This next question asks the same things. But there are two different methods that could be used.
Method 1: The equation is in the form $y=a x^{2}+b x+c$. We could convert it to the form $f(x)=a(x-h)^{2}+k$ by completing the square. Then we would pick out the vertex and other information.

Method 2: The equation is in the form $y=a x^{2}+b x+c$. So calculate the $x$-value of the vertex as $x=\frac{-b}{2 a}$. How would you find the $y$-value that goes with it?
expl 3: Find the vertex and axis of symmetry for this quadratic function. Determine if the vertex is a minimum or a maximum. Graph the function.

$$
f(x)=
$$

So, answer the following questions:
a.) What is the vertex of the parabola?

b.) What is the axis of symmetry?
c.) Is the vertex a maximum or minimum value? What is the extremum value?
d.) Graph the function.

## Increasing and Decreasing Functions:



Notice how a parabola changes from increasing to decreasing or vice versa at its vertex.
expl 4: Use the function in the previous example. Find the range of the function. Find the intervals over which the function is increasing or decreasing.

$$
f(x)=
$$

expl 5: A projectile is fired straight upward, with a muzzle velocity of 100 feet per second. The height $h(x)$ of the projectile (in feet) is given by $h(x)=-16 x^{2}+100 x+25$ where $x$ is the number of seconds after the projectile is fired.
a.) How long does it take the projectile to get to its maximum height?
b.) Find the maximum height of the projectile.
c.) During what values of $x$ is the projectile going up and during which values of $x$ is the projectile going down? In other words, give the intervals where the function is increasing and decreasing.
d.) What is the height when $x$ is 5 seconds?

expl 6: A cereal company has determined that the revenue $R(x)$ that they make from selling $x$ boxes of cereal is given by $R(x)=500 x-6 x^{2}$. If they make (and then sell) $x$ boxes, their cost is given by $C(x)=700+8 x$. Find a formula for the company's profit if they make and sell $x$ boxes. How many should they make and sell to maximize their profit? What is the maximum profit?


This is an optional application which is very cool but will not be represented in the homework.
expl 7: A rock is dropped from a high cliff. The sound of it hitting the ground is heard 2.5


