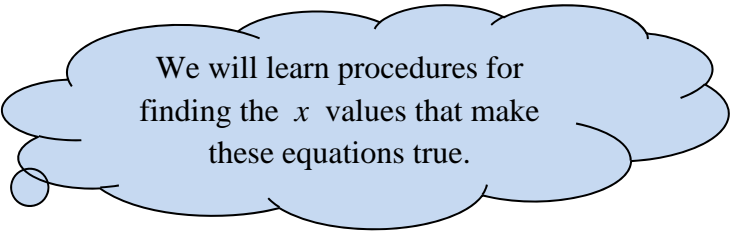


College algebra

Class notes

Solving Rational and Radical Equations (section 3.4)



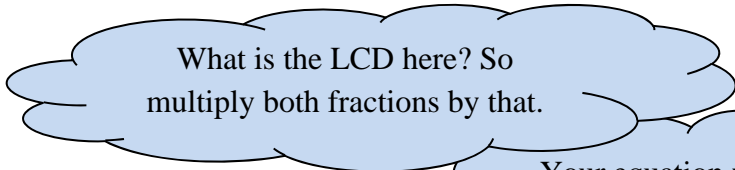
We will learn procedures for finding the  $x$  values that make these equations true.

### Algebraically Solving Rational Equations:

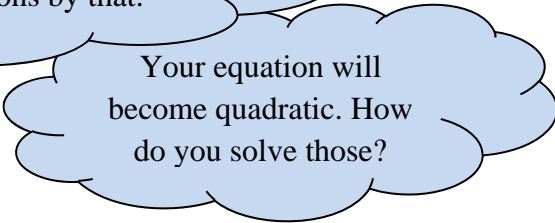
**Main idea:** The most efficient method is to multiply all fractions by their LCD. That eliminates the fractions as you will see. Make sure you always multiply **every term** by the LCD.

expl 1: Solve.

$$\frac{x-3}{x+9} = \frac{6}{x+2}$$



What is the LCD here? So multiply both fractions by that.



Your equation will become quadratic. How do you solve those?

### Checking your answer:

An important step in solving rational equations is checking your answer. Sometimes a solution turns out to be what is called an “**extraneous solution**”. Even though you get it by doing good algebra, it does **not** make the equation true after all. Remember, the whole point is to find the value(s) of the variable that make the equation true.

### Cross-multiplying:

Notice the last problem quickly became  $(x+2)(x-3) = 6(x+9)$  once we multiplied by the LCD. We were then able to solve it like any quadratic equation. That step can be short-cut by what is commonly called cross-multiplying.

When you have an equation with just one fraction on each side, you can “cross-multiply” to get a simpler equation with no fractions. But beware! It only works if the equation is in the form “one fraction = another fraction”.

expl 2: Solve.

$$\frac{x}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$$

Factor all  
bottoms to find  
the LCD.

Remember these?

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Check  
your  
solutions!

expl 3: Solve.

$$\frac{1}{x-6} - \frac{1}{x} = \frac{6}{x^2-6x}$$

When good algebra gets  
you to an equation that  
is true no matter what  $x$   
is, what do you  
conclude about the  
original equation?

...But why can't we  
say the solution is  
"all real numbers"?

expl 4: Solve.

$$\frac{7}{2x+6} - \frac{3}{x^2-9} = \frac{5}{x-3}$$

Factor all  
bottoms to find  
the LCD.

### Algebraically Solving Radical Equations:

**Main idea:** Addition undoes subtraction, division undoes multiplication. What undoes square roots? When your variable is buried under a radical sign, what do you do to unbury it?

expl 5: Solve.

$$\sqrt{4x-3} - 5 = 0$$

What undoes  
square rooting?

Square root 16 and get 4.  
What do we do to 4 to get  
back to the 16?

Isolate the  
radical  
first!

Check  
your  
solutions!

expl 6: Solve.

$$\sqrt{x+16} + 4 = x$$

Check your answers  
whenever you  
square both sides of  
an equation.

expl 7: Solve.

$$\sqrt{3x+5} = -7$$

How do you know there is  
no solution just by looking  
at the equation?

Would that be the case if  
it were  $\sqrt[3]{3x+5} = -7$ ?

**Literal Equations (Formulas):** Consider the formula in the next example. If we know  $A$  and  $P$  and want to find  $I$ , the formula is just what we need. However, if we know  $A$  and  $I$  and want to find  $P$ , this formula can be more work than we want. Imagine if we had many such problems. If we solved the formula for  $P$ , it would help out a lot.

expl 8: Solve  $I = \sqrt{\frac{A}{P}} - 1$  for  $P$ .

Isolate  $P$ .

expl 9: Solve  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$  for  $R_2$ .

**Solving equations graphically:** The simplest way to solve an equation graphically is to graph  $y = \text{“left side of equation”}$  and  $y = \text{“right side of equation”}$  and see where they intersect. Let’s take another look at some of the equations we just solved. Draw a quick, labeled graph and plot and label the solutions.

expl 10: Graphically solve.

$$\sqrt{x+16} + 4 = x$$

**Optional Worksheet: Roots and Intersections on your Calculator (82, 83, 85, 86):**

Here we explore finding roots (commonly called zeroes or  $x$ -intercepts) of a single function and the intersection of two functions. We use these skills when we solve equations graphically.

Instructions for the 83 will work for the 84 too.