What are the possible outcomes when you flip three coins? How do you know you have them all?

General Education Mathematics
Class Notes
Counting: Introduction to Counting Methods (Section 12.1)

There are so many great counting problems. How many five-card poker hands are possible from a 52-card deck? If I flip four coins, I could get HHHT or HHTT or HTHT. What else? How many possibilities are there? What if I flip ten coins? If four of my ten friends want an invite to my exclusive birthday celebration, how many ways can I make up a four-person guest list?

We will learn some fundamental rules when counting this stuff. We will look at tree diagrams and how they help us count, and more. We will work on imagining a tree diagram in lieu of actually drawing it. Why would we do that?

Let's get started.

expl 1: How many outcomes are possible when you do the following things? Also, list out the outcomes. (Use abbreviations.) Use set notation with the lovely, curvy set brackets, { and }.

a.) flip a single coin

b.) flip two coins

c.) flip three coins

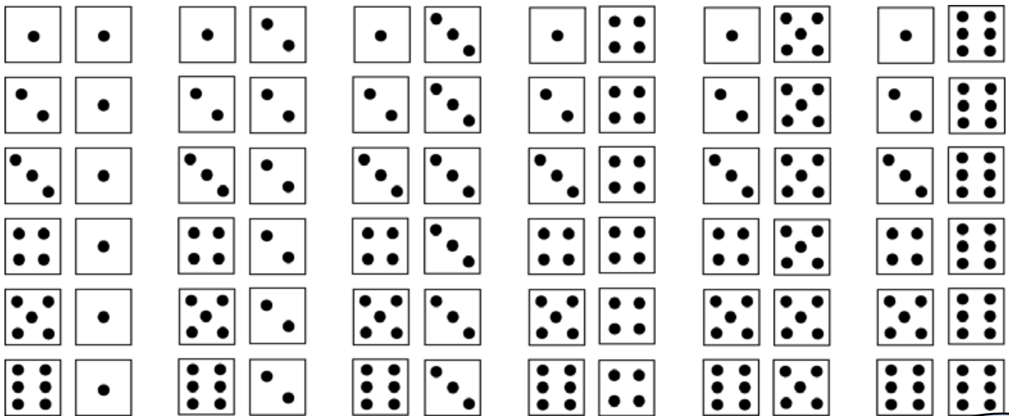
d.) roll a single die

Die is the singular version of dice.

e.) roll a pair of dice (We'll list these outcomes on the next page; just give the number of outcomes.)

Outcomes for Rolling Two Distinguishable Six-sided Dice:

This set-up happens often so let's look at an organized list and then a tree diagram. We can picture our dice outcomes this way. (This listing of outcomes is the **sample space**.)



Notice this generates 36 outcomes.

You may be told you have a red die and a green die. That makes them distinguishable.

Let's draw a tree diagram. Start on the far left of the page at a single point near the middle of the space you have. Draw six segments from this single point, labeling them 1 through 6 for the first die. Next, from each end, draw another six segments, labeling each 1 through 6 for the second die. Do you see the outcomes above displayed?

Tree Diagrams Imagined:

Tree diagrams are great but they can get cumbersome quickly. Imagine if we had just *three* dice or two *twenty-sided* dice. You can see how drawing the tree diagram can be too much, but that does *not* stop you from imagining it!

expl 2: Describe the tree diagram *without* drawing it. How many outcomes would we have in each case?

a.) Roll three six-sided dice.

b.) Roll two *twenty-sided* dice.

c.) Flip six coins.

d.) Select a two-digit number, with repetition allowed, from the digit set S = {1, 2, 3, 4, 5}. Doubles, like 22, are allowed.

Counting the outcomes in the previous example was a matter of multiplying the number of tree branches for the first die (or coin or selected number or etc.) and the number of tree branches for the second die (or coin or selected number or etc.) and number of tree branches for the third die (or coin or selected number or etc.) and this goes on until the end.

This is, in fact, what we see as the **Fundamental Counting Principle** which we study in the next section. Before we get there, though, let's investigate more counting problems that are *not* so cut and dry.

expl 3: How many outcomes would we have in each case?

a.) Select two letters, with repetition allowed, from the set S = {A, B, C, D, E}. Doubles, like BB, are allowed. However, we will consider BA to be the same as AB.

> We will go step by step. How many doubles are there? Now, imagine the tree diagram that finds the many combinations like AB (but *not* AA). How many combinations do we get there? Did we overcount them?

b.) Consider rolling two distinguishable, six-sided dice. (Refer to the sample space given earlier.) How many ways can we get a sum of 10 on the dice? List the ways out.

expl 4: You have four friends (Abby, Bob, Cathy, and Doug) you invited to a movie. You will seat the friends in a row. How many ways can this be done? Let's complete this tree diagram.

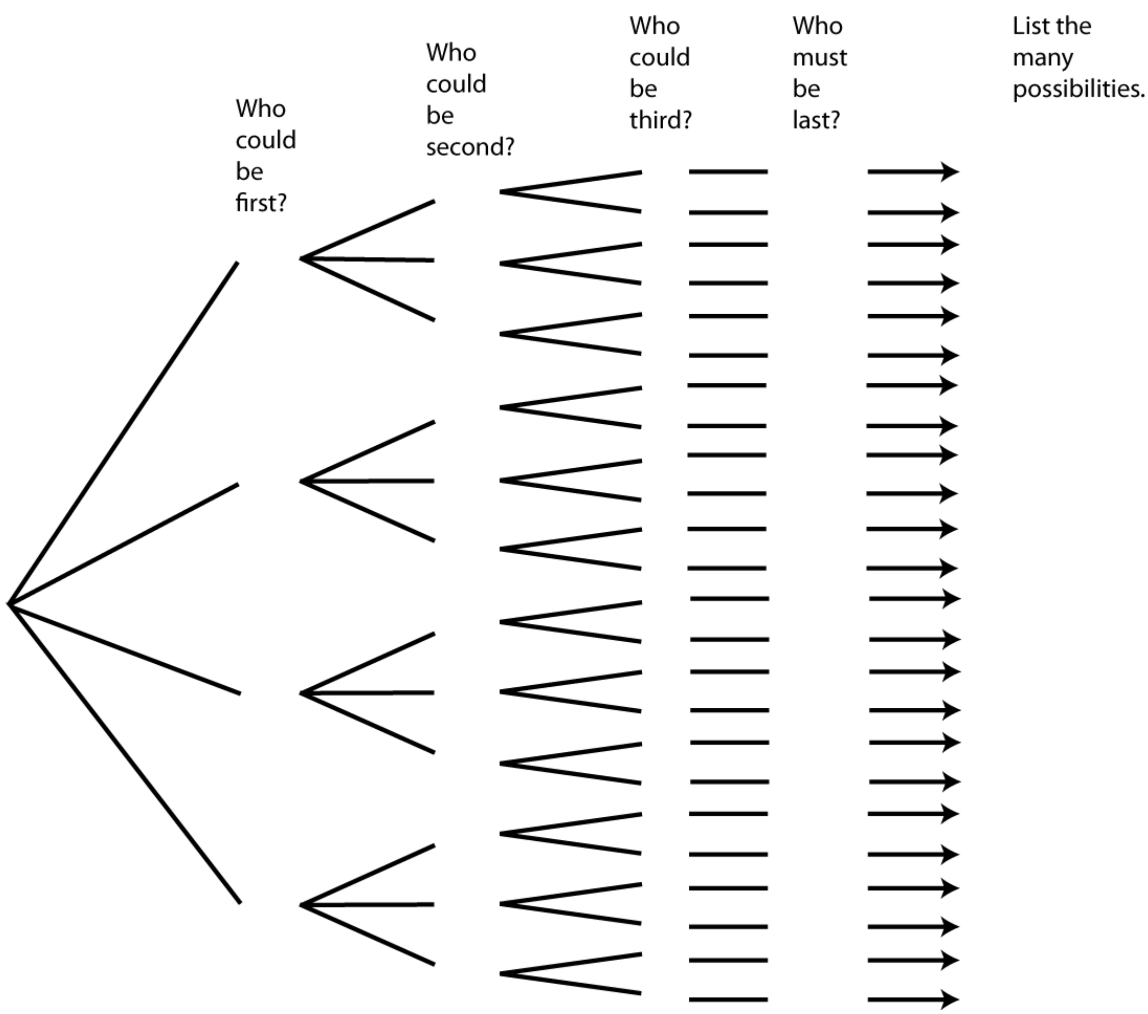
a.) The first (left-most) branching decides who gets seated first. Label them A, B, C, and D.

In the next section, we will see how the **Fundamental Counting Principle** gets us this answer. However, here we will use a tree diagram.

b.) From the end of each branch, we assume that person has already been seated and we go on to seat the next friend. Do you see why there are three branches for the second friend? Write the possibilities down in the tree diagram.

c.) Again, we assume the first two friends have been seated and we move our attention to the third friend. Write the possibilities down in the tree diagram.

d.) You will notice that placing the first three friends also decides the very last. So, from the end of each branch, place the last friend. This gives us the complete tree diagram. Follow each branch from left to right and record (on the far right) the various ways you can seat your friends.



expl 5: *Imagine* the tree diagram that we would have in the previous example if we had 7 friends. How many seating arrangements would we have then?

expl 6: Radio Shack sells a line of mix-and-match stereo systems. They sell three different kinds of speakers, two different receivers, and four different music players. If Billy Bob wants a stereo (including speakers, a receiver, and a music player), how many systems are possible for him? Imagine the tree diagram.

expl 7: I have three students I need to meet. They are each available according to the schedule below. Use a tree diagram to draw out my possible meeting schedules.

Sharri	Monday at 9:00; Tuesday at 10:00; Wednesday at 9:00
Mark	Monday at 9:00; Tuesday at 12:00
Tom	Monday at 10:00; Tuesday at 10:00

Do *not* draw a branch if it would double-book me. Or, cross off all double-bookings at the end.

Start with Sharri on the left side. Each new branching will be for another student.

expl 8: A local ice cream parlor offers three flavors, cherry, chocolate, and peach. Count the number of possible three-scoop cones in each situation. I leave you room to draw tree diagrams if you want.

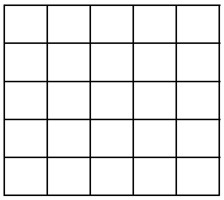
a.) You order a three-scoop cone. The flavors can be repeated or not, and two cones are different if they have the same flavors but in different orders. How many three-scoop cones are possible?

b.) You order three scoops. The flavors can be repeated or not, and two cones are different if they have the same flavors but in different orders. You will *not* eat peach ice cream cause you think peaches look like butts. You will also *not* eat the cone if it has only one flavor. How many cones are available to you now?

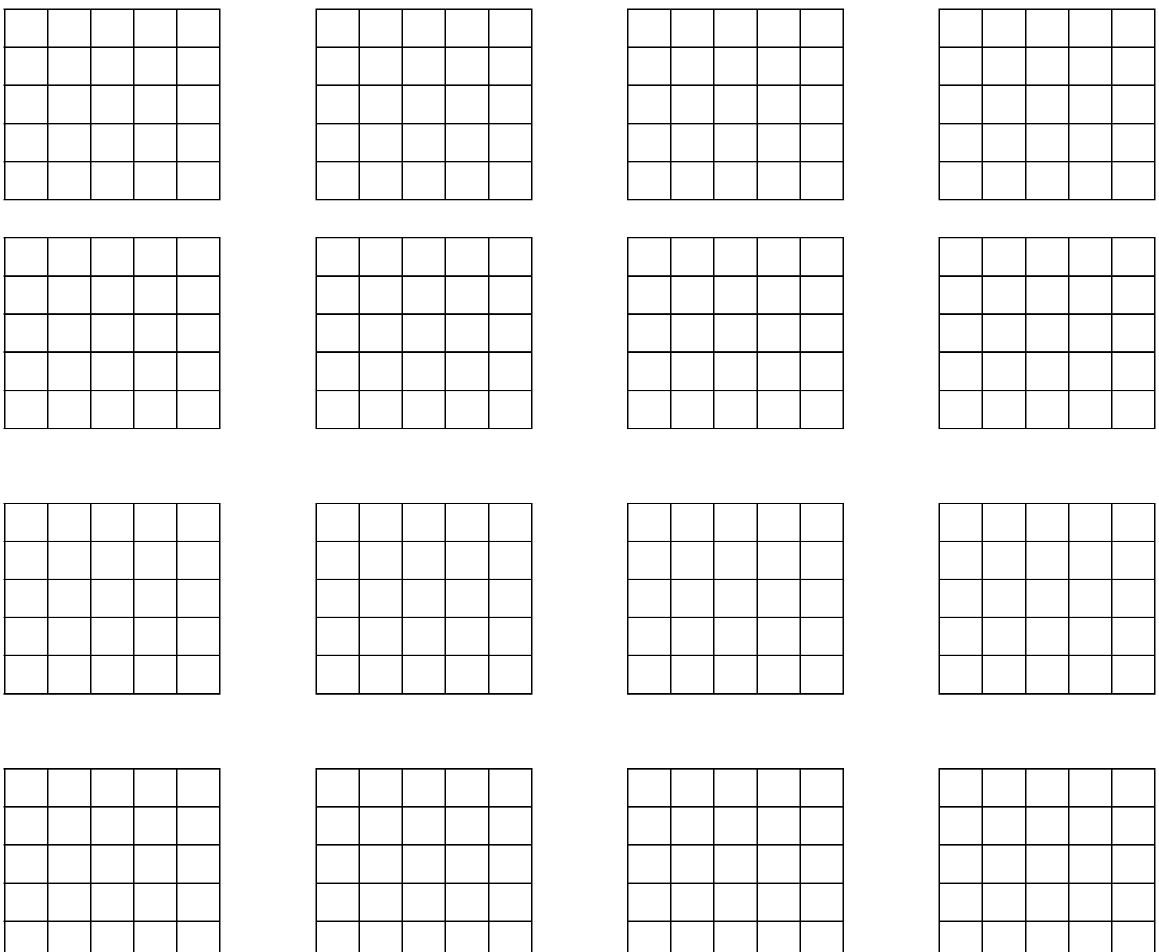
If a homework problem states to count cones with only cherry and chocolate, do *not* count the cones with *only one* of those flavors.

Drawing Geometric Figures in Counting Problems:

expl 9: Pictured below is a 5x5 checkerboard. How many 4x4 squares are possible within it? How many 3x3 squares are possible? Use the extra boards to draw out all possible 4x4 and 3x3 squares.



A 3x3 square is defined to be 3 rows by 3 columns.



Draw a single 3x3 on each board. There are extra boards.