"Bob is tall" is true and "Jill is short" is false. So, what can we say about the statement "Bob is short or Jill is short"?

General Education Mathematics
Class Notes
Logic: Statements, Connectives, and Quantifiers (Section 3.1)

We will see connections between what we did in Set Theory and what we do here. Also, we will have lots of notation and new terminology, so use the last page of these notes to fill out a reference sheet to be used throughout the chapter.

In symbolic logic, we only care whether statements are true or false – *not* their content.

Definition: A **statement** in logic is a declarative sentence that is either true or false. We represent statements by lowercase letters such as p, q, or r.

Examples of statements: Remember that to be a statement, the sentence must be either true or false; however, *we do not need to know which it is.*

a) AIDS is a leading killer of women.

b) If you eat less and exercise more, you will lose weight.

c) In the last 10 years, we have reduced by 25% the amount of greenhouse gases in the atmosphere.

d) Garth Brooks or Taylor Swift had the highest earnings in the entertainment industry in 2015.

Counterexamples of statements: Can you tell why these are *not* statements?

e) Come here.

f) When did dinosaurs become extinct?

g) This statement is false.

This is a paradox. *It cannot be either true or false.* If we assume that this sentence is true, then we must conclude that it is false. On the other hand, if we assume it is false, then we conclude that it must be true.

For the statements *a* – *d*, notice they either contain a single idea or more than one idea.

Definitions: A **simple statement** contains a single idea. A **compound statement** contains several ideas combined together. The words used to join the ideas of a compound sentence are called **connectives**. The most common connectives are *not*, *and*, *or*, *if…then*, and *if and only if*.

expl 1: Which of these sentences are statements? Do *not* worry if you do not *know* if it is true or false.

> Is the sentence declaring something? Is it either true or false?

a.) Jeannie is going to Asia this winter.

b.) Matt, please go.

c.) If you want this bike, then pay me $500.

expl 2: Identify each statement as simple or compound. If it is compound, identify the connectives used.

> There are five categories of connectives: negation, conjunction, disjunction, conditional, and biconditional. Look for the words *not*, *and*, *or*, *if…then*, and *if and only if*.

a.) Jeannie is going to Asia this winter.

b.) I have a dream.

c.) Lori or Mika will visit you if you are sick.

d.) If you want this bike, then you'll pay me $500.

e.) If you show me yours, I'll show you mine.

f.) Bob and Carlos will graduate with me.

g.) Bob will graduate with me.

h.) The shape I have drawn is a rhombus if and only if it is a polygon and it has four equal sides.

Five Categories of Connectives: Negation, Conjunction, Disjunction, Conditional, and Biconditional:

We use the connective words *not*, *and*, *or*, *if…then*, and (to a lesser extent) *if and only if* in our everyday language. Let's give these notions more concrete definitions.

Definition: A **negation** is a statement expressing the idea that something is *not* true. We use the symbol $\sim$ to represent negation. (This is the tilde, found to the left of 1 near the top of your keyboard.)

Pronounce $\sim p$ as "not p".

expl 3: Let's call j the statement "Jeannie is going to Asia this winter." How would you write, in words, the negation $\sim j$?

Definition: A **conjunction** expresses the idea of "*and*". We use the symbol $\wedge$ to represent a conjunction.

Pronounce $p \wedge q$ as "p and q".

expl 4: Consider the following statements.

p: The tenant pays utilities.

d: A \$150 deposit is required.

a) Express the statement "It is *not* true that: the tenant pays utilities and a \$150 deposit is required" symbolically.

Consider commas and colons when determining order.

b) Write the statement $\sim p \wedge \sim d$ in words.

Use parentheses to denote order.

What is the difference between the statements in parts *a* and *b*?

Definition: A **disjunction** conveys the notion of "*or*". We use the symbol $\vee$ to represent a disjunction.

> Pronounce $p \vee q$ as "p or q".

expl 5: Consider the following statements.

h: We will build more hybrid cars.

f: We will use more foreign oil.

Write the following statement symbolically.

We will not build more hybrid cars or we will use more foreign oil.

Definition: A **conditional** expresses the notion of "*if . . . then*". We use the arrow $\rightarrow$ to represent a conditional.

> Pronounce $p \rightarrow q$ as "if p, then q".

expl 6: Let's define b to be the statement "you want this bike" and p to be the statement "you'll pay me \$500". Express the conditional "If you want this bike, then you'll pay me \$500." symbolically.

expl 7: For the following conditional, define statements p and q such that the sentence reads $p \rightarrow q$.

If wishes were fishes, we'd all swim in riches.

Definition: A **biconditional** represents the idea of *"if and only if"*. We use a double arrow $\leftrightarrow$ to denote this symbolically. This double arrow implies that the logic flow goes both ways.

This one feels weird because we do *not* use it much in normal English. However, we can think of it as *both* "if p, then q" and *also* "if q, then p".

Pronounce $p \leftrightarrow q$ as "p if and only if q".

This essentially means that p and q are saying the same things.

Definitions are written in this form, although we do *not* usually see the phrasing *"if and only if"*. The compound statement from earlier, "The shape I have drawn is a rhombus if and only if it is a polygon and it has four equal sides.", is really the definition that would be normally written as "A rhombus is a polygon that has four equal sides." Do you hear the implied phrase *"if and only if"*?

expl 8: Write the biconditional statements in symbolic form. Use letters to represent the statements and define them specifically. Use the symbol $\sim$ for negation.
a.) Congress will *not* pass new energy legislation or we will develop alternative energy sources, if and only if global warming will *not* increase.

b.) A group of pus-filled bumps forming a connected area of infection under the skin is defined as a carbuncle.

Be sure your statements are really "statements".

Universal and Existential Quantifiers:

In addition to connectives, there are other special words called **quantifiers** that we will use. They tell us "how many" and fall into two categories, **universal** and **existential**.

Definition: Universal quantifiers are words such as *all* and *every* that state that all objects of a certain type satisfy a given property.

Examples of Universal Quantifiers:

> *All* citizens over age 18 have the right to vote.
>
> *Every* triangle has an interior angle sum of 180 degrees.
>
> *Each* NASCAR driver must register for the Daytona 500 by August 1.

Definition: Existential quantifiers are words such as *some*, *there exists,* and *there is at least one* that state that there are one or more objects that satisfy a given property.

Examples of Existential Quantifiers:

> *Some* drivers qualify for reduced insurance rates.
>
> *There is* a number whose square is 25.
>
> *There exists* a bird that cannot fly.

expl 9: Identify each quantifier and label it as universal or existential.

a.) Some hamburgers are delicious.

b.) Every rational number can be written as a fraction.

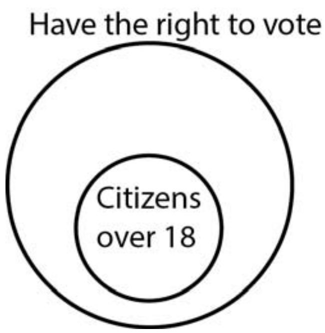
c.) All dogs go to heaven.

Is the opposite "Some dogs go to heaven" or "No dogs go to heaven"?

Negating Statements with Universal Quantifiers:
How do we negate a statement like, “All citizens over age 18 have the right to vote”?

We start by tacking on the word *not* at the beginning of the sentence. However, we do *not* leave it there but, rather, we rephrase the sentence too. Let’s look at this visually with what is called a **Euler Diagram**.

This is the Euler diagram for the statement
“All citizens over age 18 have the right to vote”.



A **Venn diagram** shows *all possible* logical relationships between a collection of sets. An **Euler diagram** only shows relationships that exist in the real world (according to the statement).
(source: https://creately.com/blog)

If we want to negate this statement, we could say “*Not* all citizens over age 18 have the right to vote”. Draw a Euler diagram for this new statement.

Are we saying that *no* citizens over age 18 have the right to vote or are we saying that *only some* citizens over 18 have the right to vote? Which feels like the opposite statement? Which does your picture show?

Quick Procedure:
When we want to negate a statement with a universal quantifier as we did above, we can replace the words “not all are” with “at least one is not” or “some are not”.

Negating Statements with Existential Quantifiers:
How do we negate a statement like, “Some drivers qualify for reduced insurance rates”?

Draw a Euler diagram of this statement. How are the circles for “reduced insurance rates” and “drivers” related?

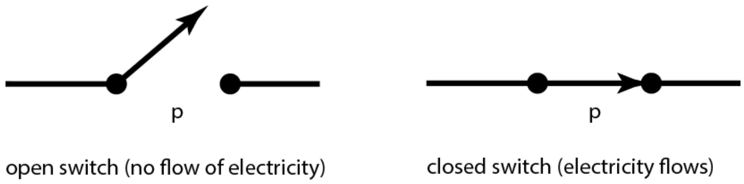
Now, rephrase the statement as, “It is false that some drivers qualify for reduced insurance rates”. What does that Euler diagram look like? Draw it. How do we move the circle for “drivers”? In normal English, how do we say that?

Quick Procedure:
When we want to negate a statement with an existential quantifier as we did above, we can replace the words “not some are” with “none are”.

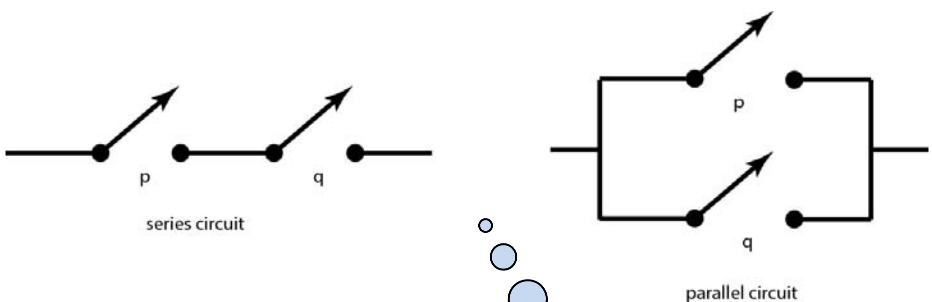
expl 10: Negate each quantified statement and rewrite it in English.
a) All customers will get a free dessert.

b) Some tablet computers have a two-year warranty.

expl 11: In 1937, Claude Shannon showed that computer scientists could use symbolic logic to design computer circuits. Electricity passes through a switch when it is closed and does *not* flow when it is open. See the pictures below.

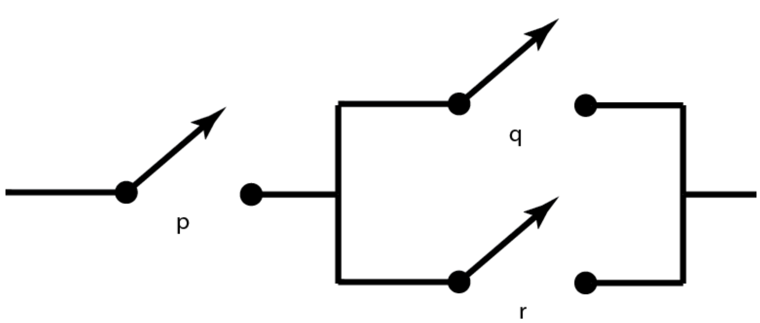


Electricity flows through a series circuit (two switches in a row, below left) if *both* switches are closed. This corresponds to the conjunction, $p \wedge q$, in logic. Electricity flows through a parallel circuit (below right) if *either* p *or* q is closed. This corresponds to the disjunction, $p \vee q$, in logic. These are shown below.



For electricity to flow through the circuit on the left, we need *both* switches closed. For electricity to flow through the circuit on the right, we need *only one* switch to be closed.

Write the logical form that corresponds to the circuit below (in order for electricity to flow).



Logic Notation Reference Sheet:

Complete this list as we proceed through chapter 3. There are extra spots in case you want to add your own items.

Symbol	**Meaning**
p, *q*, etc. (lower case letters)	a declarative sentence called a statement that is either true or false
$\sim p$	not *p*; negation of the statement *p*
$p \wedge q$	*p* and *q*; a conjunction (meaning "and")
$p \vee q$	*p* or *q*; a disjunction (meaning "or"); in math, we mean one *or* the other, *or* possibly both
$p \rightarrow q$	if *p*, then *q*; a conditional
$p \leftrightarrow q$	if *p*, then *q* and also if *q*, then *p*; a biconditional (meaning "if and only if")
T	true; a statement is true
F	false; a statement is false
M	maybe; a statement may be true or false; used in three-valued logic
$q \rightarrow p$	the ______________ of $p \rightarrow q$ (fill in blank)
$\sim p \rightarrow \sim q$	the ______________ of $p \rightarrow q$ (fill in blank)
$\sim q \rightarrow \sim p$	the __________________ of $p \rightarrow q$ (fill in blank)
$\therefore$	