Mr. Pin is a poet. All poets have carbuncles. Therefore, Mr. Pin has carbuncles.

General Education Mathematics
Class Notes
Logic: Verifying Arguments (Section 3.4)

We will look at a whole argument, some statements and a conclusion, to see if it is a valid argument. What does it mean for an argument to be valid, anyway?

Definitions: An **argument** is a series of statements called **premises** followed by a single statement called the **conclusion.** An argument is **valid** if whenever all the premises are true, then the conclusion must also be true.

We will use truth tables to evaluate the validity of an argument. Notation will play a large role in what we do and how we think through these problems.

expl 1: State whether the argument is valid. We will work this step by step.

Premises: If news on inflation is good, then stock prices will increase.
News on inflation is good.

Therefore, stock prices will increase.

Shorthand:
$p \to q$
p
$\therefore q$

a.) The premises could be thought of as "$p \to q$ *and* p". The conclusion is "q". Define, in words, these statements p and q.

b.) The argument has the form $[\,(p \to q) \;\wedge\; p\,] \to q$. We will make a truth table for this argument. If the truth table shows that whenever the premises are true that you will also have a true conclusion, then we will say the argument is valid. Complete the truth table.

p	q	$p \to q$	p	q
T	T			
T	F			
F	T			
F	F			

In future, you can omit rows where the two premises are *not both* true.

c.) Look for the row(s) of the table where *both* premises are true. There may be more than one row. Is the conclusion also true? If so, the argument is valid. Is the argument valid?

Procedure for Verifying Arguments:

1. Make a truth table with separate columns for each premise and the conclusion.

2. Examine *only* the lines in the table in which all of the premises are true.

3. If the conclusion is also true for all of the lines you examined in step 2, then the argument is **valid**.

4. If the conclusion is false for *even one* of the lines you examined in step 2, then the argument is **invalid**.

Arguments get a lot more complicated than example 1. Let's try out some more.

expl 2: State whether the argument is valid. We will work this step by step.

Premises: If Phillippe joins the basketball team, then he will *not* be able to work part-time.
Phillippe did *not* join the basketball team.
____
Therefore, he will be able to work part-time.

Remember, use *not* as a connective, not within a statement.

a.) Using the statement definitions below, write the argument in symbols.

b: Phillippe joins the basketball team.
w: Phillippe will be able to work part-time.

b.) Make a truth table. Fill in the column headings you need.

b	***w***				
T	T				
T	F				
F	T				
F	F				

Find *all* of the rows where the premises are both true.

c.) What is your conclusion? Is the argument valid?

expl 3: State whether the argument is valid. We will work this step by step.

Premises: If a movie is exciting, then it will gross a lot of money.
This movie grossed a lot of money.

Therefore, this movie is exciting.

You can use the notation $\therefore$ to mean therefore.

a.) Using the statement definitions below, write the argument in symbols.

e: This movie is exciting.
m: This movie grossed a lot of money.

b.) Make a truth table. Fill in the column headings you need.

e	***m***			
T	T			
T	F			
F	T			
F	F			

c.) What is your conclusion? Is the argument valid?

Known Fallacies to Avoid:

Both of the arguments in examples 2 and 3 proved invalid. In fact, they are examples of well-known **fallacies**, or bits of faulty reasoning. We must be on the look-out for them.

Fallacy of the Inverse:	**Fallacy of the Converse:**
$p \to q$ $\sim p$ $\therefore \sim q$	$p \to q$ q $\therefore p$

Recall, $\therefore$ means therefore.

Valid Arguments / Laws:

Many valid arguments will fall into one of the four types below.

Which did we see in example 1?

Law of Detachment:	**Law of Contraposition:**
$p \to q$ p $\therefore q$	$p \to q$ $\sim q$ $\therefore \sim p$
Law of Syllogism:	**Law of Disjunctive Syllogism:**
$p \to q$ $q \to r$ $\therefore p \to r$	$p \vee q$ $\sim p$ $\therefore q$

Do you see why these must always be valid?

expl 4: Which law from above is used in this argument?

Remember, use *not* as a connective, not within a statement.

Premises: If it’s brown, then we will flush it down.
It was *not* flushed.

Therefore, it is *not* brown.

Use p and q to denote the statements of the argument and define them specifically in words. Write the argument in symbols to show its similarity to a law.

expl 5: Which law from above is used in this argument?

Premises: Brooke or John will attend the meeting.
John will *not* attend the meeting.

Therefore, Brooke will attend the meeting.

Use p and q to denote the statements of the argument and define them specifically in words. Write the argument in symbols to show its similarity to a law.

expl 6a: Supply a conclusion that will make this argument valid.
If you exercise each day, then you will have more energy. You do *not* have more energy.

Therefore, ____.

Compare the premises with the laws from the last page.

expl 6b: Which law did you use to complete the argument above?

expl 7: Determine if this form represents a valid argument.

Argument:

$$\begin{array}{c} \sim p \\ p \wedge q \\ \hline \therefore\ q \wedge p \end{array}$$

Get out your truth table.

It is possible that the premises are *never both* true. This is a **valid but unsound** argument.

Three Variables:

expl 8: Let's investigate this argument in three variables. Is it valid?

Argument:

$$\begin{array}{c} p \\ q \to \sim p \\ q \to (r \wedge p) \\ \hline \therefore\ r \end{array}$$

Again, we are looking for rows with all true premises. What is the truth value for r is those rows?

p	q	r	$\sim p$	$r \wedge p$	$q \to \sim p$	$q \to (r \wedge p)$	r
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

I repeat the r column for convenience.

For the argument to be valid, what do we need? Is this argument valid?

Arguments given in words can be distilled down to the shorthand notation and then tested with a truth table.