How likely is a sum of 7 when you roll a pair of dice? How likely is a lightning strike?

General Education Statistics
Class Notes
Introduction to Probability (Section 5.1)

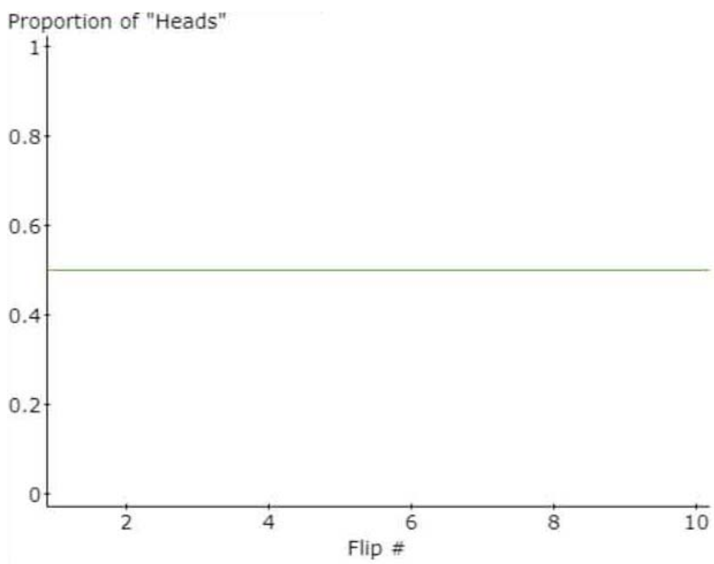
Consider rolling two dice. How likely is it that you get two even numbers? Or a sum of seven? How likely is it that a family of three children has exactly two boys? What is the likelihood of getting struck by lightning? We will study these and other probability questions.

expl 1: We will use the applet "Simulating the Probability of a Head with a Fair Coin" available on MyMathLab. To find the applet from the MML page, select Multimedia Library and search for Applets from Chapter 5.

Since we are using the computer to "flip" coins, we call this a simulation.

a.) What does it mean to have a *fair* coin?

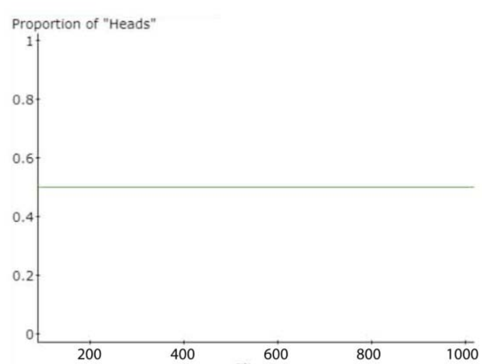
b.) Have the applet simulate *one* toss to start you off. Use the graph here to record whether it was a heads or tails. Notice the scale of the horizontal axis (labeled "Flip number").



c.) Have the applet simulate *five* more tosses. Continue your graph above to record the tosses.

d.) At the end of six tosses, what is your "proportion of heads"? What does that mean? How many heads out of six tosses did you have?

e.) What does the horizontal line on the graph indicate?



f.) Now, have the applet toss 1000 coins. What happens? Roughly draw the graph here. Record the proportion of heads too.

Definition: Probability is a measure of the likelihood of a random phenomenon or chance behavior occurring.

Probability deals with experiments that yield random short-term results or **outcomes**, yet reveal long-term predictability. The long-term proportion in which a certain outcome is observed is the **probability** of that outcome.

We will see a Very Useful Formula but first let's define some terminology.

Definition: In probability, an **experiment** is any process that can be repeated in which the results are uncertain. Perhaps, that is rolling a die or tossing a coin. Each time we do the experiment counts as a **trial**.

Definition: The **sample space,** S, of a probability experiment is the collection of all possible outcomes. We will use set notation with the { and } brackets.

Definition: An **event** is any collection of outcomes from a probability experiment. An event may consist of one outcome or more than one outcome. Examples are rolling a six on a die or getting two heads when tossing two coins. We will denote events with one outcome, sometimes called **simple events**, e_i. In general, events are denoted using capital letters such as E. The probability of event E is denoted by $\boldsymbol{P(E)}$.

Definition: A **success** is considered to be an event we are interested in. In example 1, a success was "heads" on the coin.

expl 2: Consider the probability experiment of having two children and recording their sexes.
a.) Identify the outcomes of the probability experiment and write them in set notation, labeled as the sample space, S.

Outcomes may be denoted e_1, e_2, e_3, and e_4.

b.) Define the event E = "have exactly one boy". In other words, which outcomes comprise this "event"? Use set notation. For the next example, these will be considered "successes".

A Very Useful Formula: Probability is the proportion (or fraction) that compares the number of successes to the number of possibilities. This assumes that the possibilities are equally likely.

$$\text{Probability of an event} = \frac{\text{number of successes}}{\text{number of total possibilities}}$$

Sometimes we use percent form.

expl 3: Return to the experiment of having two children. What is the probability of having exactly one boy? Use the formula above.

Now considering this formula, we can deduce some **Fundamental Rules of Probabilities**. Let's think through them together.

a.) Can the probability of an event be negative? Explain.

b.) Can the probability of an event be more than 1 (or 100%)? Explain.

c.) What is the sum of the probabilities of all possible outcomes? Complete these tables below to answer this question.

These tables are **probability models**.

Coin Toss	
Outcome	**Probability**
heads	
tails	
total	

Use your simulated coin toss from example 1 (six tosses).

Family with two children	
Outcome	**Probability**
boy, boy	
girl, girl	
boy, girl	
girl, boy	
total	

Use your sample space from example 2.

Now, what do you think this sum will always be?

Types of Probabilities:
There are actually three distinct types of probability.

The **Classical Method** (also called Theoretical probability) was used when we looked at the probability of having exactly one boy with two births. **We say that the outcomes are equally likely**. This means that each outcome (remember these were boy/boy, girl/girl, boy/girl, and girl/boy) are all equally likely to occur as any other. The Very Useful Formula on the last page technically uses this model. This method uses counting techniques to figure the number of successes and possibilities.

The **Empirical Method** (also called Experimental probability) was used when we actually tossed a coin a number of times as we did in example 1. We actually do the experiment (or simulate it) and simply count the number of successes. The formula really is the same but some people like to rewrite it like this.

$$\text{Probability of an event} = \frac{\text{number of successes}}{\text{number of trials}}$$

Then there are **Subjective Probabilities**. You see this when a weather forecaster tells you that there is a 35% chance of rain tomorrow. Horse betting, another example, is based on probabilities that a certain horse will win. These probabilities are determined by personal judgement. The probability is *not* based on a probability experiment or counting equally likely outcomes.

Definitions: If an event is **impossible**, the probability of the event is 0. If an event is a **certainty,** the probability of the event is 1. An **unusual event** is an event that has a low probability of occurring (usually lower than 0.05 or 5%, but that number is *not* set in stone).

expl 4: Consider the **probability model** to the right that shows the probabilities of selecting each color in an M&M bag.
a.) Verify that it is a proper probability model. That is, make sure it adheres to the Fundamental Rules of Probabilities discovered on page 3.

Are the probabilities all between 0 and 1 (inclusive)? Do they sum to 1?

Color	**Probability**
Brown	0.11
Yellow	0.15
Red	0.12
Blue	0.24
Orange	0.23
Green	0.15

b.) Which is the most likely color to select from the bag? Why?

expl 5: What does it mean for the probability of an orange M&M to be 0.23? Label each sentence as true or false.

__________ A. If you select 100 M&M's, you would expect to get around 23 orange ones.

__________ B. Getting an orange M&M would be considered an unusual event.

__________ C. If you select 100 M&M's, you should get exactly 23 orange ones.

__________ D. Getting an orange M&M would be considered a certain event.

__________ E. Getting an orange M&M should happen about one-quarter of the time.

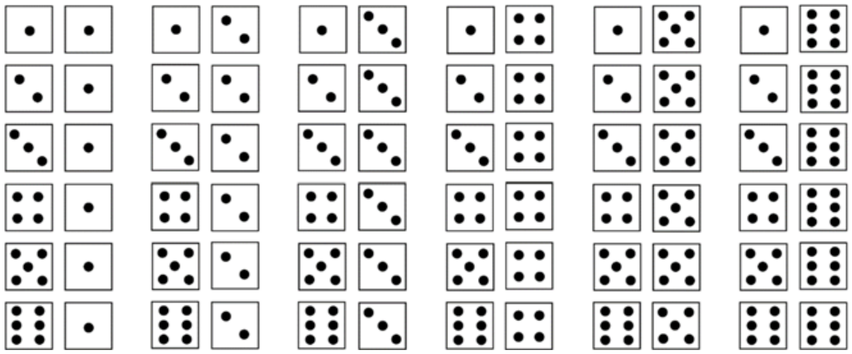
__________ F. An orange M&M is more likely than a yellow M&M.

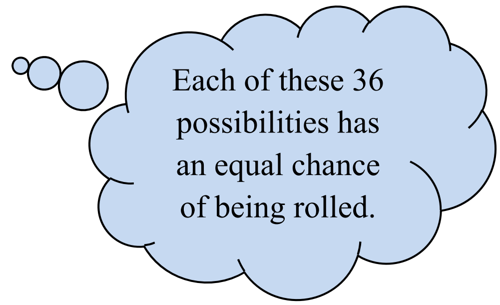
Think back to the last part of example 1. Recall that the proportion of heads got closer and closer to 0.50 or 50% as the number of trials increased. This is an example of the Law of Large Numbers.

Definition: The Law of Large Numbers:
As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to the true probability of the outcome.

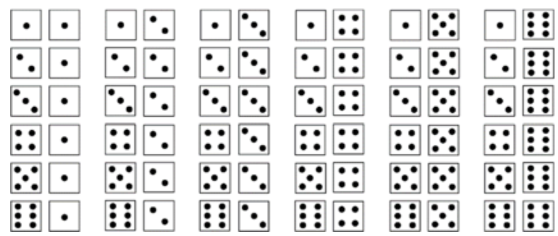
> You should *not* expect to get exactly 23 orange M&M's given 100 randomly selected M&M's. However, as we pull more and more M&M's from a very large bag, the proportion of orange M&M's should get closer and closer to 23%.

expl 6: Consider rolling a pair of distinguishable six-sided, fair dice. The sample space is shown below. Do you see why this is the sample space? Answer the questions that follow.

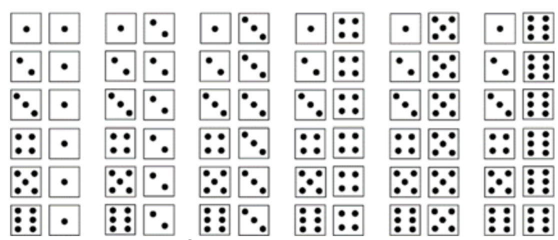




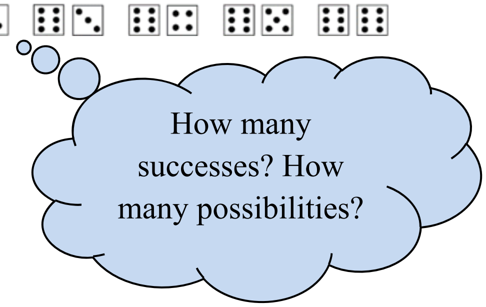
a.) What is the probability of rolling a pair (meaning the two dice display the same number)? Circle the possibilities in the sample space below that would be considered "successes".

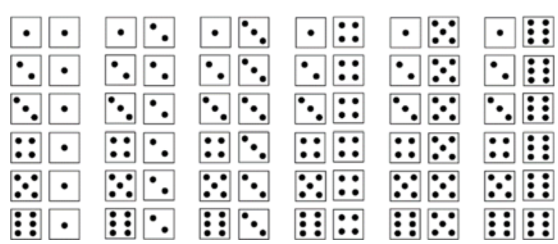


b.) What is the probability of rolling a sum of seven? Circle the possibilities in the sample space below that would be considered "successes".



c.) What is the probability that both dice turn up as even numbers? Circle the possibilities in the sample space below that would be considered "successes".





expl 7: A class has four students (Amanda, Bailey, Christie, and Doug) who want to team up to give an extra-credit report. A two-person team will be chosen at random. Answer the following questions.

a.) Determine the sample space. In other words, list all possible two-person teams from the people A, B, C, and D. Enclose your sample space in set brackets {…} and label it as *S*.

b.) Are all of the possible two-person teams equally likely to be chosen?

c.) What is the probability that Amanda is chosen?

d.) What is the probability that Amanda is *not* chosen?

e.) What is the probability that Doug and Christie are *both* chosen?

How Dangerous is Lightning?

Lightning is a major cause of storm related deaths in the U.S. A lightning strike can result in a cardiac arrest (heart stopping) at the time of the injury, although some victims may appear to have a delayed death a few days later if they are resuscitated but have suffered irreversible brain damage.

According to *the NWS Storm Data*, over the last 30 years (1987-2016) the U.S. has averaged 47 *reported* lightning fatalities per year. Only about 10% of people who are struck by lightning are killed, leaving 90% with various degrees of disability. More recently, in the last 10 years (2007-2016), the U.S. has averaged 30 lightning fatalities.

Odds of Becoming a Lightning Victim (based on averages for 2007-2016)				
Estimated U.S. population as of 2017				325,000,000
Average Number of Deaths Reported	30	Estimated number of Injuries	270	300
Odds of being struck in a given year (estimated total deaths + estimated injuries)				1/1,083,000
Odds of being struck in your lifetime (Est. 80 years)				1/13,500
Odds you will be affected by someone struck (10 people for every 1 struck)				1/1,350

(source: http://www.lightningsafety.noaa.gov/odds.shtml)

expl 8: How do you think they got the figure 1/1,083,000 for the odds of being struck by lightning in a given year? (Hint: Look for a rounded answer.)

They are using the term "odds". The concept of odds is related to probability but slightly different. We will not cover odds per se.

expl 9: Out of the 300 people in a large lecture class, 159 of them have their own car on campus.

a.) If we select a person at random from the class, what is the probability that they will have their own car?

b.) If we survey 1000 similar people, how many should we expect to have their own car?