If you roll two dice, does getting a 2 on one die influence the other die's result?

General Education Statistics
Class Notes
Independent Events and the Multiplication Rule (Section 5.3)

In a three-child family, does the first two children being boys make it more or less likely that the third child is a boy? If we pull two cards out of a poker deck, does the first coming out red make it more or less likely that the second one is red too? This is the idea of independence which we explore here. We will also look at probabilities such as the likelihood a three-child family has *at least* two boys.

**Definition:** Two events $E$ and $F$ are **independent** if the occurrence of event $E$ in a probability experiment does *not* affect the probability of event $F$. Two events are **dependent** if the occurrence of $E$ does, in fact, affect the probability of event $F$.

expl 1: Consider each experiment and pair of events. Which do you think are independent?

a.) Experiment: Pull two cards from a deck. Do *not* replace the first card before pulling the second.

Event $A$: First card is a face card

Event $B$: Second card is a face card

b.) Experiment: Roll two distinguishable, six-sided dice. One die is red; one is white.

Event $A$: Red die is 4

Event $B$: White die is an even number

c.) Experiment: Pull two cards from a deck. Replace the first card before pulling the second.

Event $A$: First card is a face card

Event $B$: Second card is a face card

d.) Experiment: Roll two distinguishable, six-sided dice. One die is red; one is white.

Event $A$: The red die is 6.

Event $B$: The sum of the two dice is 6.

If event $A$ happens, does it change the likelihood of event $B$ happening?

expl 2a:What is the probability that a 29-year-old has high cholesterol? What is the probability that a 29-year-old, who eats daily at fast food restaurants, has high cholesterol? Does the fact that he eats daily at fast food restaurants influence the probability that he has high cholesterol? If yes, then the events are *not* independent. What two ”events” are we discussing?

expl 2b: If we select, at random, two 29-year-olds, does one having high cholesterol make it more likely that the second has high cholesterol? In other words, would they be independent? What if we did *not* select them at random, but rather chose two men who were from the same family?

expl 3: We'll consider the experiment and events explored in example 1*b*. It is copied here.

Experiment: Roll two distinguishable, six-sided dice. One die is red; one is white.

Event $A$: Red die is 4

Event $B$: White die is an even number

a.) Find $P(A)$.

b.) Find $P(B)$.

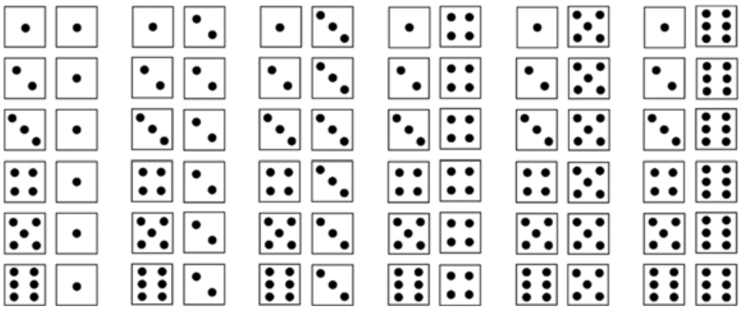
c. ) Calculate $P(A) \cdot P(B)$.

**What does AND mean?**

You may recall we defined what OR meant in the last section of notes. We said that we use OR in math a little differently than in English. When we talk about event $E$ OR event $F$, we mean either $E$ or $F$ occurs, or *possibly both*, occur.

When we talk about event $E$ AND event $F$, we mean that *both* events occur simultaneously.

expl 3d: Now, returning to the previous experiment and considering the sample space given below, take the first die to be red and the second die to be white. Circle those outcomes that are in *both* events $A$ *and* $B$.



Event $A$: Red die is 4

Event $B$: White die is an even number

What is the probability $P(A \text{ AND } B)$? Do you notice anything about this value and what we got in the last example? This leads to a rule.

**Multiplication Rule for Independent Events:**
If $E$ and $F$ are independent events, then $P(E \text{ and } F) = P(E) \cdot P(F)$.

Also, if you know that $P(E \text{ and } F) = P(E) \cdot P(F)$, then you can say $E$ and $F$ are independent.

You can extend this to many independent events.
**Multiplication Rule for $n$ Independent Events:**
If $E_1$, $E_2$, $E_3$, … and $E_n$ are independent events, then we know that
$P(E_1 \text{ and } E_2 \text{ and } E_3 \ldots \text{ and } E_n) = P(E_1) \cdot P(E_2) \cdot P(E_3) \cdot \ldots \cdot P(E_n)$.

**Optional Worksheet: Mutually exclusive and independent events:**
What was your reasoning with example 1*d*? How did you know they were dependent? This worksheet explores the two ideas of mutually exclusive events and independence and how they co-mingle.

**Finding Probabilities involving the phrase "at least":**
Often these questions seem easier when looking at them as "complement" problems.

expl 4: Approximately 13% of the population is left-handed. Two people are selected at random. Answer the following questions. Here we will assume that all people are either left- or right-handed. We will *not* worry about ambidextrous people (the self-righteous snobs).

a.) What is the probability that both people chosen are left-handed?

b.) Write out the sample space $S$ for this experiment, labeling a left-handed person as $L$ and a right-handed person as $R$.

So, $RL$ would mean a right-handed person is picked followed by a left-handed person.

Are these outcomes equally likely?

c.) What is the probability that at least one person chosen is right-handed?

This is the complement of both being left-handed. Do you see why?

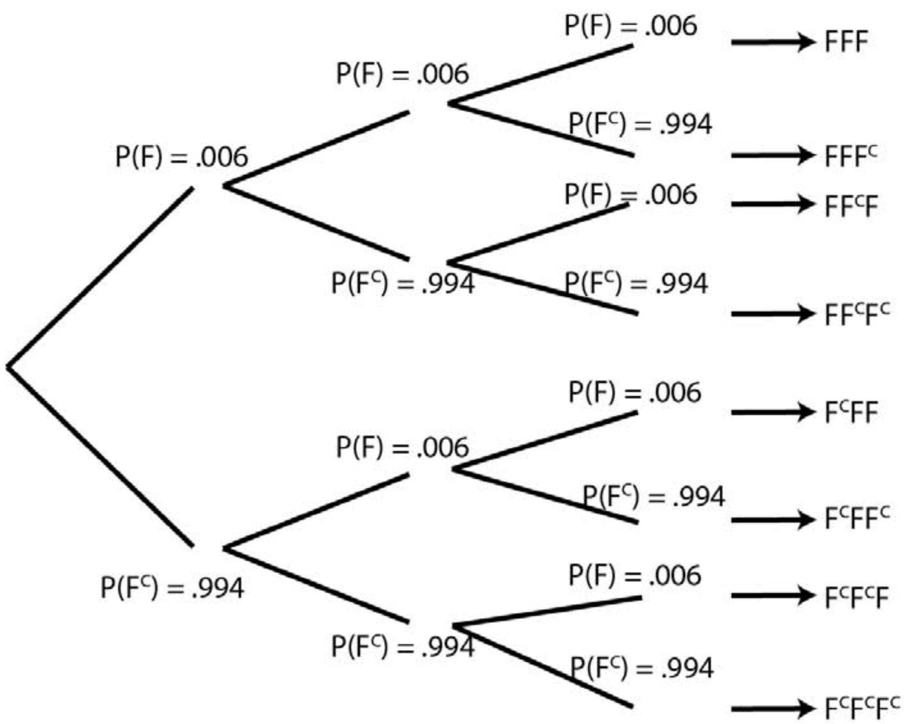
d.) What is the probability that both people chosen are right-handed?

If $P(\text{left-handed}) = 0.13$, then what is $P(\text{right-handed})$?

expl 5: In airline applications, failure of a component can result in catastrophe. As a result, many airline components utilize something called triple modular redundancy. This means that a critical component has two backup components that may be utilized should the initial component fail. Suppose a certain critical airline component has a probability of failure of $0.006$ and the system that utilizes the component is part of a triple modular redundancy.

a.) Assuming each component's failure or success is independent of the others, what is the probability that *all three components fail*, resulting in disaster for the flight?

b.) Find and mark the event considered in part *a* on the tree diagram below. (The symbol $F$ stands for failure and $F^C$ stands for "*not* failure".)




c.) What is the probability *at least* one of the components does *not* fail? Find and mark the outcomes considered here. (Remember that $F^C$ stands for "*not* failure".)

How are the events in parts *a* and *c* related?

expl 6: Eliza has a 64% probability of making a free-throw in basketball practice. Assume all free-throws are independent of one another. Answer the following questions.

a.) What is the probability that Eliza makes three free-throws in a row?

b.) What is the probability that Eliza makes three free-throws in a row, but does *not* make four in a row?

This would look like
“make, make, make, miss”.

c.) If Eliza makes three attempts, what is the probability that she misses *at least* one?

**Worksheet: The difference between OR and AND:**
We explore events involving OR and AND to better understand what is meant. We will see how using the sample space to organize “successes” is useful and efficient. A problem utilizes a two-way table for practice with those.

expl 7: According to a poll, about 14% of adults in a country bet on professional sports. Data indicates that 45.8% of the adult population in this country is male. Answer the following questions.

a.) Are the events "male" and "bet on professional sports" mutually exclusive? Explain.

b.) Assuming that betting is independent of gender, compute the probability that an adult from this country selected at random is a male *and* bets on professional sports.

You have a formula for this.

c.) Using the result in part *b*, compute the probability that an adult from this country selected at random is male *or* bets on professional sports.

You have a formula for this.

d.) The poll data indicated that 8.3% of adults in this country are males *and* bet on professional sports. What does this indicate about the assumption in part *b*?

e.) How will the information in part *d* affect the probability you computed in part *c*? Recalculate the probability that an adult from this country selected at random is male *or* bets on professional sports, given the more accurate information given in part *d*.