Knowing a formula for dy/dx helps us graph the function y.

Differential Equations
Class Notes
Direction Fields (Section 1.3)

When our diff. eq. is a formula for the unknown function's dy/dx, we can use it to graph the "slopes" of the function at various points. From that graph, we can fill in the function itself.

Definition: Direction field for a diff. eq.: a plot of short-line segments drawn at various points in the xy-plane showing the slope of the solution curve (the unknown function) at these points. They are also called **slope fields**.

We will use them to draw specific solution curves for initial value problems.

Consider the first-order diff. eq. $dy/dx = f(x, y)$ for some function $f(x, y)$ in x and y.

Recall y is a function that is the *solution* to this diff. eq. and dy/dx is the slope of this function.

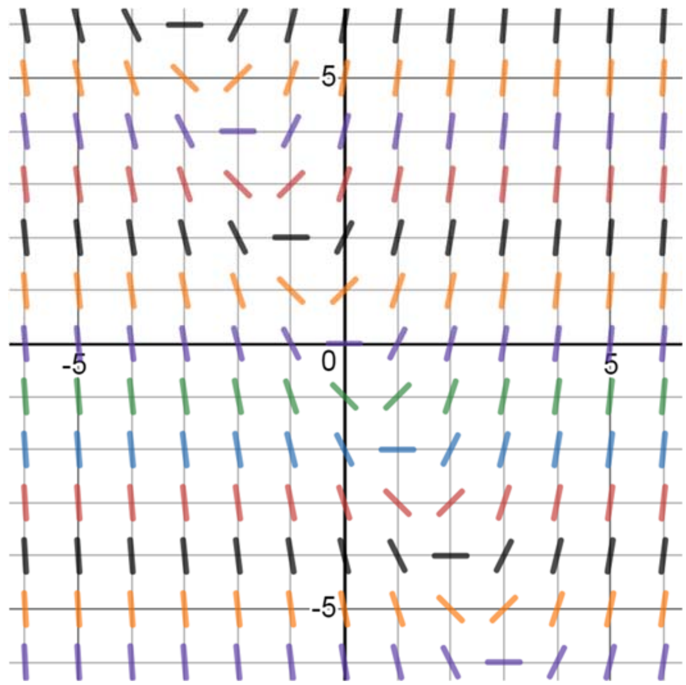
Think of $dy/dx = 2x + y$. What would $f(x, y)$ be?

For instance, if $dy/dx = 2x + y$, we know at some point, say the point (2, 3), the solution (the function y) must have a slope of $dy/dx = 2x + y$.

What is the slope of the unknown function at (2, 3)?

expl 1: The direction field of $\frac{dy}{dx} = 2x + y$ is drawn below. Answer the questions.

a.) Sketch the graph of the solution curve that passes through $(0, -2)$. From this sketch, write an equation of the solution.



(source: https://www.desmos.com/calculator/p7vd3cdmei)

"If a solution goes through a point on the slope field, then its slope at that point would match the segment on the slope field."

--Khan Academy

Later, we will know this diff. eq. as linear and could solve it to indeed get this solution.

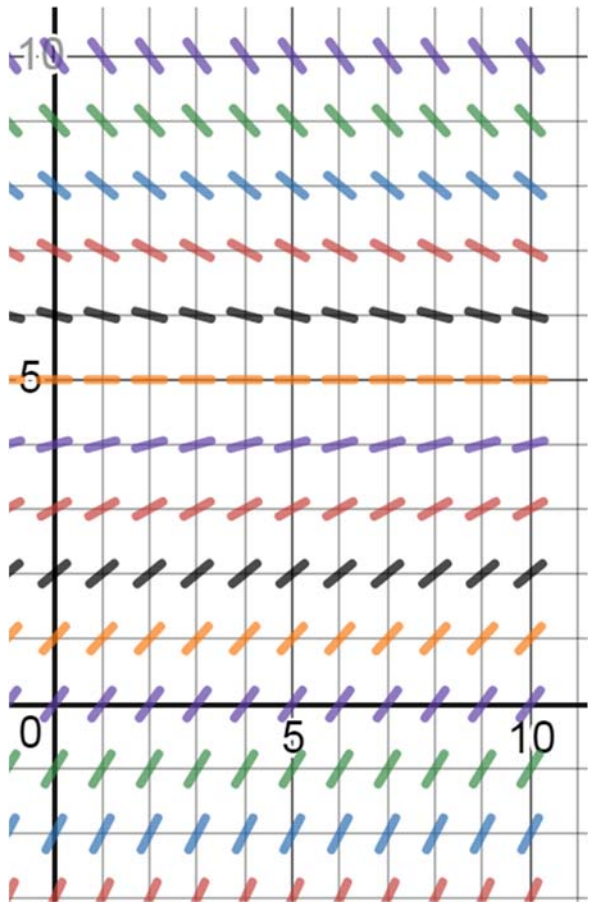
b.) Sketch the graph of the solution curve that passes through $(-1, 3)$. You do *not* need to figure the solution's equation.

c.) What can we say about the solution in part *b* as $x \to \infty$ and as $x \to -\infty$? In other words, $\lim_{x \to -\infty} y = \lim_{x \to \infty} y = ?$

expl 2: A model for the velocity v at time t of a certain object falling under the influence of gravity in a viscous medium is given by the equation $\frac{dv}{dt} = 1 - \frac{v}{5}$. From the direction field given, sketch the solutions with initial conditions $v(0) = 3$, 5, and 10. Take note of the scale used. Do *not* worry about units such as feet/second.

Why is the value $v = 5$ called a "terminal velocity"?

Start your sketch over for each value of v.



(source: https://www.desmos.com/calculator/p7vd3cdmei)

expl 3: The logistic equation for the population (in thousands) of a certain species is given by $\frac{dp}{dt} = 3p - 2p^2$.

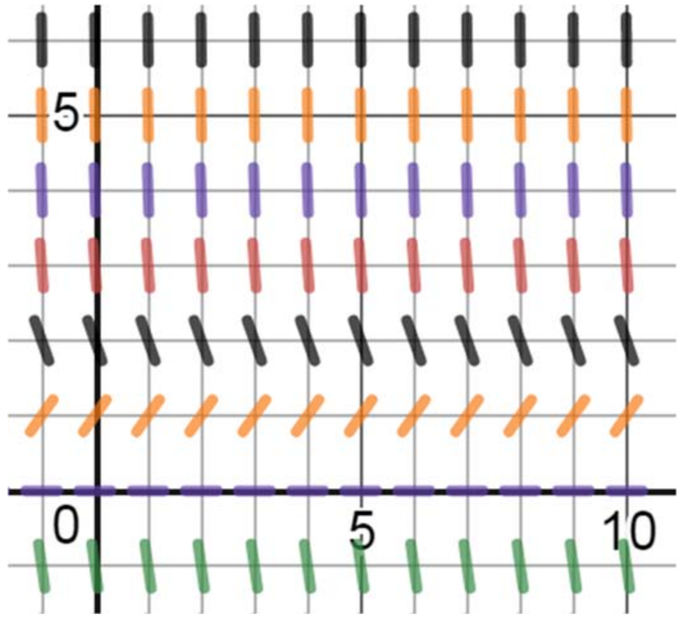
a.) Sketch the direction field by using a computer program. In class, we will use www.desmos.com.

The book also covers the method of isoclines, which we skip.

expl 3 continued: The logistic equation for the population (in thousands) of a certain species is given by $\frac{dp}{dt} = 3p - 2p^2$. Here is the direction field.

b.) If the initial population is 3000 (that is, $p(0) = 3$), what can we say about the limiting population or $\lim_{t \to \infty} p(t)$?

c.) If $p(0) = .8$, what is $\lim_{t \to \infty} p(t)$?



(source: https://www.desmos.com/calculator/p7vd3cdmei)

d.) Can a population of 2000 ever decline to 800? Explain.

Worksheet: Direction Fields Worksheet:
This worksheet will give you practice sketching solution curves on direction fields and interpreting the results.