Let's throw some stuff around and see how it moves!

Differential Equations
Class Notes
Newtonian Mechanics (Section 3.4)

Imagine, if you will, some object in some motion due to some force. Aah, now that's an imagination! Good job.

Definition: Newtonian (Classical) Mechanics: the study of the motion of ordinary objects (larger than an atom, slower than the speed of light).

We will focus on forces that do *not* depend on the object's location. Only time and velocity will influence the forces acting on our object. The book goes through a simple derivation for the following diff. eq. we will use as the basis of many problems.

Newton's Second Law:

Consider an object of mass m with a force $F(t, v)$ acting on it. Let v represent the object's velocity and t be time. We know that $m \cdot \frac{dv}{dt} = F(t,v)$.

Here, $\frac{dv}{dt}$ would be the object's acceleration.

Definition: Inertial reference frame: a reference frame in which an undisturbed body moves with constant velocity.

General Procedure for Problems:

1. Determine *all* relevant forces and draw a diagram.

2. Choose an appropriate axis or coordinate system. It must be an inertial reference frame. Usually, we use *negative* for forces that act in the *opposite* direction of motion.

In contrast, you *may* be used to downward forces always considered negative.

3. Apply Newton's Second Law to determine the equation of motion for the object. Specifically, we can find $v(t)$ by solving the diff. eq. and then integrating it to find the object's position function $x(t)$.

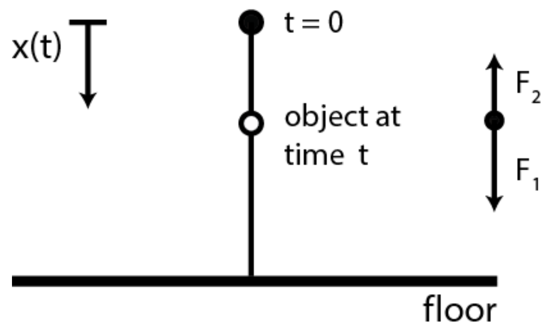
4. What, me worry?

Gravity and Air Resistance:

We'll use $F_1 = mg$ for the force due to **gravity**. Here, m is the **mass** of the object and g is the **gravitational acceleration** (taken to be 32 feet per second squared or 9.81 meters per second squared).

We'll use $F_2 = -bv$ for the force due to **air resistance**. Here, b is a positive real number called the **proportionality constant** and v is the **velocity** of the object.

An object is pictured right before a fall at time $t = 0$. It is released and falls under gravity. As it falls, air resistance pushes the object upward. We will let $x(t)$ be the distance the object has fallen at time t.



We were using $F(t, v)$ to denote the forces on the object. So, we now have

$$m \cdot \frac{dv}{dt} = F(t, v)$$

$$m \cdot \frac{dv}{dt} = F_1 + F_2$$

$$m \cdot \frac{dv}{dt} = mg - bv$$

The formulas here for $v(t)$ and $x(t)$ are very useful for certain problems.

If we are told an initial velocity v_0, we can solve this as an initial value problem, getting

$v(t) = \frac{mg}{b} + \left(v_0 - \frac{mg}{b}\right) e^{-bt/m}$ and $x(t) = \int v(t)\,dt$. When we find this integral, we see that

$$x(t) = \frac{mg}{b} t + \frac{m}{b}\left(v_0 - \frac{mg}{b}\right)\left(1 - e^{-bt/m}\right).$$

Notice that $\lim_{t \to \infty} v(t) = \frac{mg}{b}$. This is called the **limiting or terminal velocity of the object**.

expl 1: An object of mass 5 kg is released from rest 1,000 meters above the ground and allowed to fall under the influence of gravity. Assuming the force due to air resistance is proportional to the velocity of the object with proportionality constant $b = 50$ N-sec/m, determine the equation of motion for the object. When will the object strike the ground?

Draw a picture. Find F_1 and F_2 and therefore $F(t, v)$.

You *could* make a diff. eq. and solve it or use the formula for $x(t)$ on page 1.

Remember $x(t)$ is the distance the object travels. So, we want t when $x = 1000$.

When solving for t, we will ignore the exponential term in $x(t)$ because it is sufficiently small.

expl 2: An object of mass 100 kg is released from rest from a boat into the water and allowed to sink. While gravity is pulling the object down, a buoyancy force of 1/40 times the weight of the object is pushing the object up (weight = mg). If we assume that water resistance exerts a force on the object that is proportional to the velocity of the object, with proportionality constant 10 N-sec/m, find the equation of motion for the object. After how many seconds will the velocity of the object be 70 meters/second? (Round to one decimal place.)

Draw a picture. Find F_1 (gravity), F_2 (water resistance), and F_3 (buoyancy). We add these forces to get $F(t, v)$.

We must solve this diff. eq. as we have no pre-solved equation for $v(t)$.

Remember $x(t)$ is the "equation of motion" and $x(t) = \int v(t)dt$. However, we use $v(t)$ to answer the last question.

(extra room)