What if we can isolate each variable and its differential on one side of the equation? Could we integrate to solve the diff. eq.?

Differential Equations
Class Notes
Separable Equations (Section 2.2)

Recall, in general, $\frac{dy}{dx} = f(x, y)$ is a first-order diff. eq.

Definition: Separable diff. eq.: If $f(x, y)$ in the equation $\frac{dy}{dx} = f(x, y)$ can be written as a function $g(x)$ (that depends solely on x) times a function $p(y)$ (that depends solely on y), then the diff. eq. is **separable**. In other words, $\frac{dy}{dx} = f(x, y)$ is separable if and only if

$$\frac{dy}{dx} = g(x) \cdot p(y).$$

Consider that we could rewrite this equation as $\frac{dy}{p(y)} = g(x) \cdot dx$. We could integrate this to get back to the solution function y. Here is our plan explicitly.

Method for Solving Separable Equations:

To solve the equation $\frac{dy}{dx} = g(x) \cdot p(y)$, rewrite it as $h(y) \cdot dy = g(x) \cdot dx$ where $h(y) = 1/p(y)$.

Then integrate both sides $\int h(y) \cdot dy = \int g(x) \cdot dx$ to get $H(y) = G(x) + C$.

Both constants of integration combined into one, $C \in \mathbb{R}$.

The last equation gives us an implicit solution to the original diff. eq..

Additional Solutions: Constant functions $y \equiv c$ such that $p(c) = 0$ are also solutions to the diff. eq. but will *not* show up in this method. We will add them on to the solution.

Find the roots of the function $p(y)$.

The symbol $\equiv$ means "defined to be equal".

expl 1: Determine if these differential equations are separable. Identify the functions h and g.

a.) $\frac{ds}{dt} = t \cdot \ln\left(s^{2t}\right) + 8t^2$

Do you remember your log rules?
$\log_a\left(t^b\right) = b \cdot \log_a t$

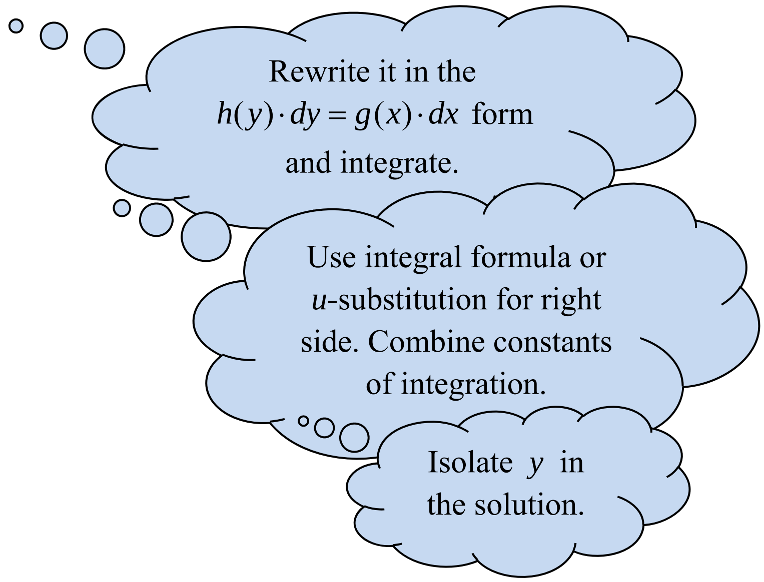
b.) $\frac{dy}{dx} - \sin\left(x + y\right) = 0$

How well do you remember trig. identities?

c.) $\left(xy^2 + 3y^2\right)dy - 2xdx = 0$

expl 2: Solve the equation.

$$\frac{dy}{dx} = \frac{x}{y^2\sqrt{1+x}}$$



After all that, let's *not* forget those possible additional solutions in the form $y \equiv c$ such that $p(c) = 0$. So what is $p(y)$ and what values of c make it zero? Do we need to amend our solution?

expl 3: Solve the equation.

$$\frac{dx}{dt} - x^3 = x$$

> Considering the switch of variables, we look for $h(x) \cdot dx = g(t) \cdot dt$.

> Do you recall partial fraction decomposition?

> Be careful with how you define the constants.

> Isolate x in the solution. It might take a bit of work.

After all that, let's *not* forget those possible additional solutions in the form $x \equiv c$ such that $p(c) = 0$. So what is $p(x)$ and what values of c make it zero?

> Sometimes, we can redefine our constant to include the additional solution.

expl 4: Solve the initial value problem.
$y' = x^3(1-y), \quad y(0)=3$

This point (0, 3) will help us solve for C in the solution.

At some point, you'll see a $|1-y|$. Near $y = 3$, what does this simplify to? Solve for y.

expl 5: **Newton's Law of Cooling:** If an object at temperature T is immersed in a medium having constant temperature M, then the rate of change of T is proportional to the difference of temperatures, $M - T$. This gives us the diff. eq. $\frac{dT}{dt} = k(M - T)$.

a.) Solve for T.

Do *not* forget about additional solutions.

b.) A thermometer reading 100° Fahrenheit is placed in a medium having a constant temperature of 70° F. After 6 minutes, the thermometer reads 80° F. What is the reading after 20 minutes?

At $t = 0$, $M = 70$ and $T = 100$. How does that help?

Find k by using the point (6, 80).

Find T(20).