

Linear function applications Solutions

NAME:

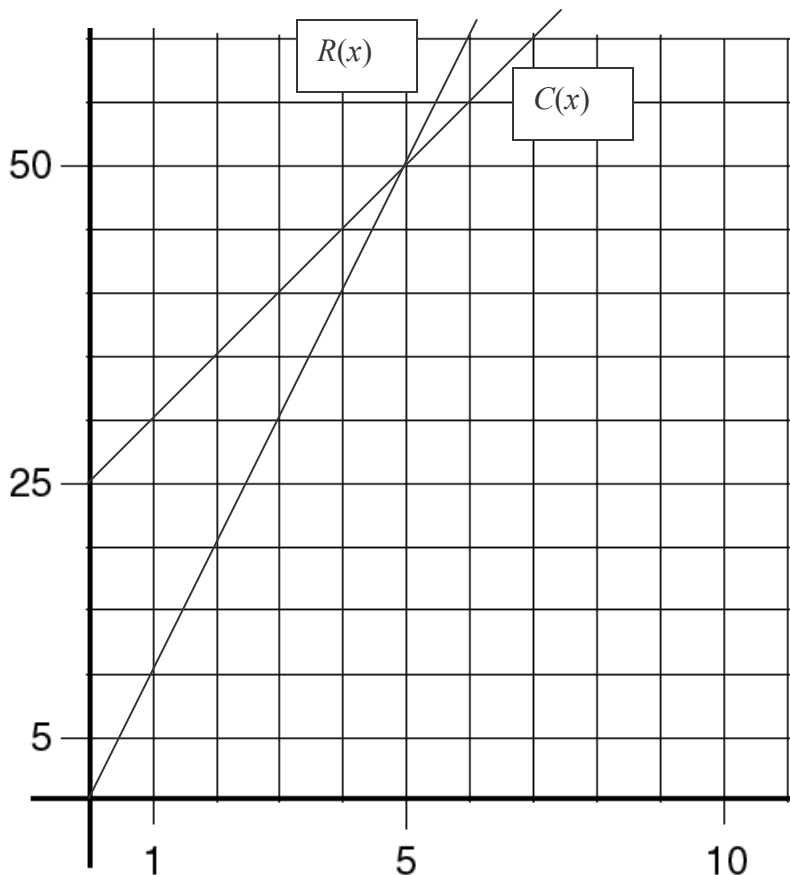
These problems involve variables that are linearly related, meaning their graphs will be straight lines. Draw the graphs using a straight edge.

1. You produce exotic candles. You have up-front costs of \$25 (perhaps the rent of your studio or one-time equipment costs) and a cost of \$5 per candle (perhaps the wax and supplies needed for each candle). You sell your candles for \$10 each.

Let x represent the number of candles you produce and sell. Determine algebraic equations for $C(x)$, the total cost, and $R(x)$, the total revenue. Use the tables to help you develop the patterns needed.

x	$C(x)$	x	$R(x)$	<i>The cost is \$25 plus \$5 for every candle. This is the money we spend.</i>
0	25	0	0	
1	$25 + 5(1) = 30$	1	$1(10) = 10$	<i>The revenue is \$10 for every candle. This is the money we bring in.</i>
2	$25 + 5(2) = 35$	2	$2(10) = 20$	
3	$25 + 5(3) = 40$	3	$3(10) = 30$	
x	$25 + 5x$	x	$10x$	

On the graph paper below, accurately graph $C(x)$ and $R(x)$. Use a straight edge.



2. Use the algebraic equations for $C(x)$ and $R(x)$ to find the cost and revenue generated if you make and sell 2 candles. (Label which is which.) Where, on the graph, is this information? Do you make or lose money?

$$C(2) = 25 + 5(2) = 35$$

$$R(2) = 10(2) = 20$$

You spend \$35 and make \$20. You lose money. This information is shown on the graph of $C(x)$ in the point (2, 35) and on the graph of $R(x)$ in the point (2, 20).

3. Use the algebraic equations for $C(x)$ and $R(x)$ to find the cost and revenue generated if you make and sell 6 candles. (Label which is which.) Where, on the graph, is this information? Do you make or lose money?

$$C(6) = 25 + 5(6) = 55$$

$$R(6) = 10(6) = 60$$

You spend \$55 and make \$60. You make money. This information is shown on the graph of $C(x)$ in the point (6, 55) and on the graph of $R(x)$ in the point (6, 60).

4. Algebraically, find the number of candles you would have to make and sell to break even, meaning you would not make or lose any money. Where, on the graph, is this information?

This means to find the x that makes cost equal to revenue. So we set cost equal to revenue and solve.

$$C(x) = R(x)$$

$$25 + 5x = 10x$$

$$25 = 5x$$

$$5 = x$$

$$x = 5$$

If we make and sell 5 candles, we will break even, meaning we will not lose or make money. This information is shown where the two functions intersect on the graph. Notice this point has an x value of 5 and a y value of 50. If you drew your graph poorly, this will be hard to see.

5. You are in the business of selling novelty t-shirts. You know that you can sell 150 if you sell them at a price of \$15. But if you lower your price to \$10, you can sell 250. Assume the relationship between price and quantity is linear. Form an equation that gives the number sold as a function of price. [Hint: Start out by writing the information given in ordered pair form (price, number sold).]

Start with the two points (15, 150) and (10, 250). Find the slope between these two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{250 - 150}{10 - 15} = \frac{100}{-5} = -20$$

Now, we know the line must be of the form “ $y = -20x + b$ ” where b is the y -intercept and (x, y) is a general point on the line. We also know that (10, 250) is on the line. So put these specific x and y values into “ $y = -20x + b$ ” and solve for b .

$$y = -20x + b$$

$$250 = -20(10) + b$$

$$250 = -200 + b$$

$$450 = b$$

Then put this b into the equation “ $y = -20x + b$ ” to get the equation “ $y = -20x + 450$ ”. This is the equation that relates number of t-shirts sold and price.

Graph this relationship. Notice price is the independent or x variable.



6a. Using the graph, estimate the price of each t-shirt if you sell 300 of them. Explain your method.

Find 300 on the y-axis and go over to the graph. Then read the x value of this point. It should be about \$7.50. This point is marked on the graph with a closed circle.

6b. Algebraically, find the price of each t-shirt if you sell 300 of them. Show work.

$$y = -20x + 450$$

$$300 = -20x + 450$$

$$-150 = -20x$$

$$7.50 = x$$

7a. Using the graph, estimate the quantity you would expect to sell if your price was \$6. Explain your method.

Find 6 on the x-axis and follow it up to the graph. Read the y value of this point. I estimate it to be about 340 or so shirts. This point is marked on the graph with an open circle.

7b. Algebraically, determine the quantity you would expect to sell if your price was \$6. Show work.

$$y = -20x + 450$$

$$y = -20(6) + 450$$

$$y = -120 + 450$$

$$y = 330$$

Notice my estimate from the graph (question 7a) was close but not exact. A more detailed graph would yield a better estimate.

The idea of questions 6 and 7 is to see the connection between algebraic and graphical means. Keep this in mind when you're solving more complicated equations.