

1. We will investigate the relationship described by the online problem “Can you figure the initial velocity of the ball?” found under the Problems section of the Website. The equation  $s = -16t^2 + v_0t + s_0$  describes the height of a bouncing ball at various times. The variable  $s$  stands for the height of the ball  $t$  seconds after release. The variables  $v_0$  and  $s_0$  stand for the initial velocity and initial height, respectively. We will use the online movie to find the initial velocity of the ball. Follow the steps outlined below.

a.) You could measure the height of the ball from anywhere on the ball, as long as you’re consistent. We choose to measure the height from the bottom of the ball. Notice the initial height is considered to be 0 feet. Write the equation  $s = -16t^2 + v_0t + s_0$  with this information substituted in.

$$s = -16t^2 + v_0t + 0$$

$$s = -16t^2 + v_0t$$

*Substitute 0 in for  $s_0$ . The point here is to start understanding what the variables represent.*

b.) Stop the ball at any time when it is not compressed. Write down the values for the height (as measured from the bottom of the ball) and the corresponding time.

$$s = 18 \text{ feet}$$

$$t = 2.8 \text{ seconds}$$

*I stopped the ball at 2.8 seconds. The height (as measured from the bottom of the ball) is 18 feet.*

c.) Substitute these values into the equation in part a and solve for the initial velocity.

$$18 = -16(2.8)^2 + v_0(2.8)$$

$$18 = -16(7.84) + 2.8v_0$$

$$18 = -125.44 + 2.8v_0$$

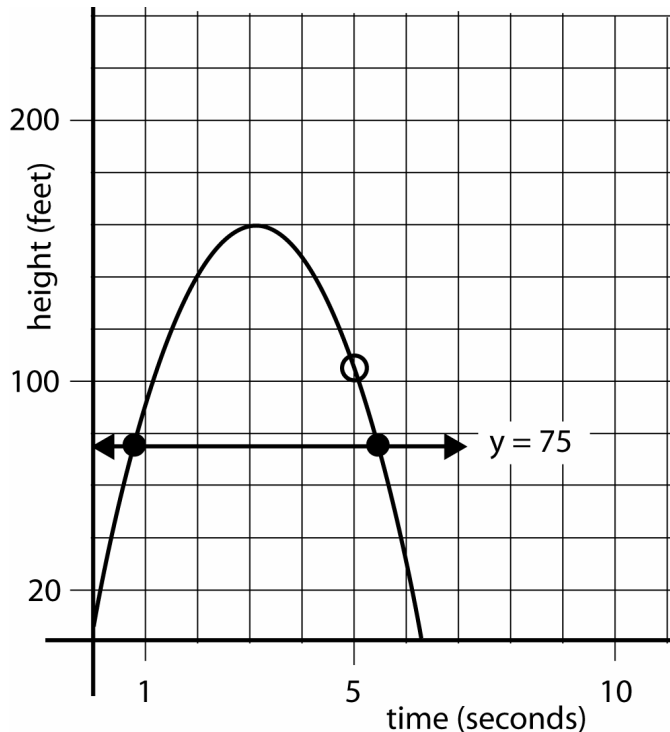
$$143.44 = 2.8v_0$$

$$51.23 = v_0$$

*Again, keep straight what the variables represent. The  $t$  is the number of seconds and  $s$  is the height at that time. Put the values from part b in there. Follow the order of operations to simplify and solve for  $v_0$ . The initial velocity of the ball is 51.23 feet per second.*

Let's work with this relationship a little more. However, we'll change the numbers involved.

2. Let's say I throw a ball straight up into the air with an initial velocity of 100 feet per second. When I let go, the ball is 4 feet high. Substitute this information into the equation  $s = -16t^2 + v_0t + s_0$ . Graph this relationship between time and height below. Remember to use the variable  $x$  when you put it into your calculator. To make your graph more accurate, use the **TRACE** button to approximate the vertex,  $y$ -intercept, and  $x$ -intercept and plot them. Then draw in the graceful curve of the graph.



Write down your equation relating height and time alongside the graph.

$$s = -16t^2 + 100t + 4$$

3. Algebraically, find the height of the ball after 5 seconds. Where is this information on the graph? Plot the corresponding point on your graph above.

$$\begin{aligned} s &= -16t^2 + 100t + 4 \\ s &= -16(5)^2 + 100(5) + 4 \\ s &= 104 \end{aligned}$$

*Put 5 in for  $t$ , solve for  $s$ . Follow the order of operations to see the height is 104 feet after 5 seconds. This is denoted on the graph by the open circle at the  $x$  value of 5. Notice its  $y$  value is 104.*

4. Graphically, find the times (number of seconds after release) when the ball is at a height of 75 feet. Write down the equation that you would need to solve. You would need to use the quadratic formula to solve it algebraically; notice solving it graphically is less nasty. Draw in the line  $y = 75$  on your graph above and show where the answers to this question appear on the graph.

$$75 = -16t^2 + 100t + 4$$

*Solve by graphing the left side, the right side, and seeing where they intersect. You will get that the ball is 75 feet high at .82 and 5.43 seconds after the ball is released. These points are marked on the graph as closed circles.*

5. Use your graph and the **Value** function (in the **Calculate** menu) to fill in the table below. (On the TI86 or 85, use the **Eval** function.)

Time (seconds)	Height (feet)
2	140
3.5	158
5	104
7.5	-146

Does your height for 5 seconds match your answer to number 3?

*Yes! The calculator's Value or Eval function does the calculation for you.*

Given the situation, does the height for 7.5 seconds make sense? Explain.

*No, a height of -146 feet indicates it burrowed into the ground 146 feet. But this did not occur. The equation  $s = -16t^2 + 100t + 4$  only applies until the ball hits the ground at about 6.29 seconds.*

6. Find the time it takes for the height to reach 100 feet. If you do this algebraically, show your work. If you do it graphically, draw a quick sketch with the parts labeled.

<p><i>Algebraically:</i></p> $100 = -16x^2 + 100x + 4$ $0 = -16x^2 + 100x - 96$ <p><i>Put into Quadratic Formula Program</i>  <i>(a = -16, b = 100, c = -96)</i></p> <p><i>Solutions x = 1.18 and 5.07.</i></p> <p><i>So the ball is 100 feet high at 1.18 and 5.07 seconds after release.</i></p>	<p><i>Graphically:</i></p>
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