Quadratic functions NAME:

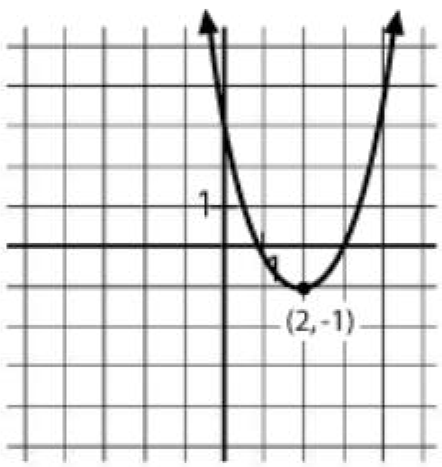
Orientation, vertex, and y-intercept **Solutions**

1. ***y*-intercept:** The y-intercept of any function is found by substituting 0 in for x. This works because the x value of every point on the y-axis is 0. You're simply using this fact along with the equation to find the y value when x is 0. So, given the general equation for a quadratic function $y = ax^2 + bx + c$, what is the y-intercept of every quadratic function?

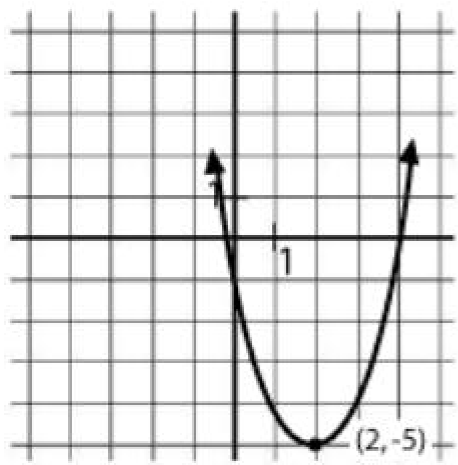
$y = a(0)^2 + b(0) + c = 0 + 0 + c = c$ *The y-intercept of a quadratic function is* c.

2. **Vertex:** Use your calculator to draw a quick graph of each quadratic function. You should try to plot the vertex and y-intercept accurately. Number 1 discussed how you can determine the y-intercept. The x-values of the vertices will be integers. You can guess them by looking at the graph. Use the **TRACE** button to approximate the y values of the vertices. Use the window shown in the graph paper below so it's easy to copy.

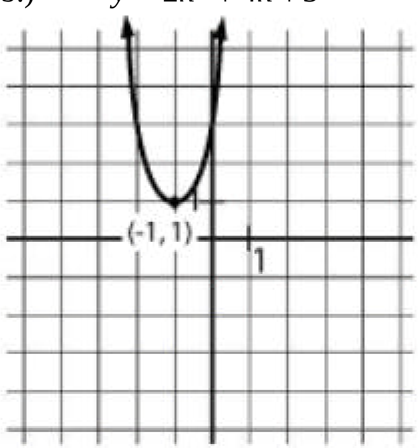
a.) $y = x^2 - 4x + 3$



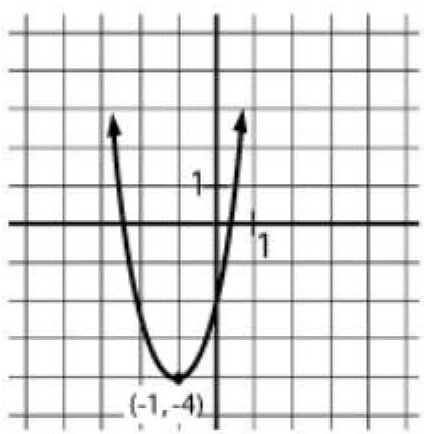
$y = x^2 - 4x - 1$



b.) $y = 2x^2 + 4x + 3$



$y = 2x^2 + 4x - 2$



3. **Vertex:** Consider the functions we just graphed in number 2. The functions in part *a* had the same *a* and *b* coefficients (as in $y = ax^2 + bx + c$). Notice this is also true of the two functions in part *b*. Also, notice the *x* values of the vertices were the same for each pair. What is the connection between the coefficients *a* and *b* and the *x* value of the vertex? Complete the table below to help organize the facts.

function	*a* coefficient	2*a* (two times *a*)	*b* coefficient	*x* value of vertex
$y = x^2 - 4x + 3$	*1*	*2*	*-4*	*2*
$y = x^2 - 4x - 1$	*1*	*2*	*-4*	*2*
$y = 2x^2 + 4x + 3$	*2*	*4*	*4*	*-1*
$y = 2x^2 + 4x - 2$	*2*	*4*	*4*	*-1*

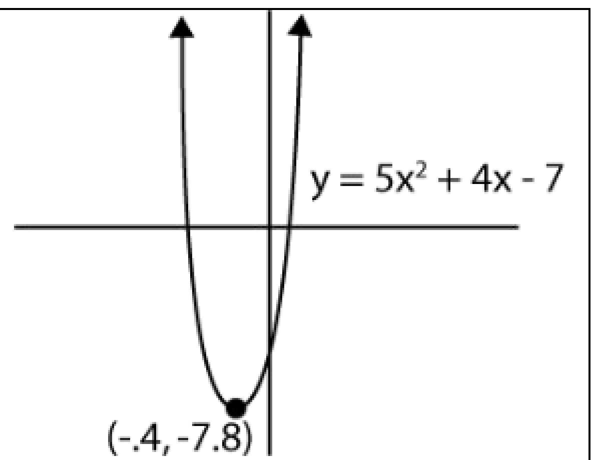
Again, what is the relationship between the *x* value of the vertex and the coefficients *a* and *b*? Let's test your theory. Make up a quadratic function. Use your theory to guess the *x* value of the vertex. Then graph the function. Copy the graph and label it. Is the vertex where you thought it would be?

This is a hard one to see. Notice for each row in the table, the x value of the vertex could be found by calculating $-b/2a$. *Whenever I have a quadratic function* $y = ax^2 + bx + c$, *the x value is always* $-b/2a$.

Let's test this theory. I made up the function $y = 5x^2 + 4x - 7$.

Its vertex should have an x value of $-4/2(5) = -4/10 = -.4$.

Sure enough, when I graph it, this is true.

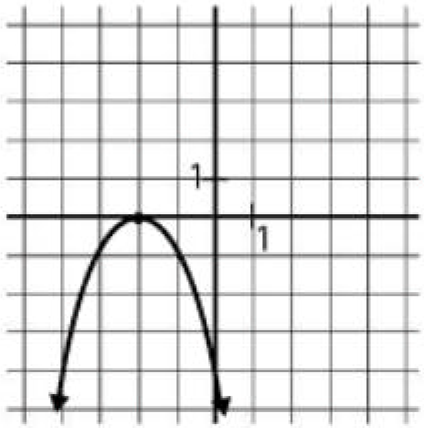


4. **Vertex:** We usually write $x = \frac{-b}{2a}$ for the *x* value of the vertex. Once I calculate this, how could I find the *y* value? As an example, without graphing, find the vertex of the function $y = 3x^2 + 18x + 4$. Write it in ordered pair notation. Quickly graph it on your calculator to check. You'll have to change the window. Use your vertex to determine a good window. Quickly copy the graph here.

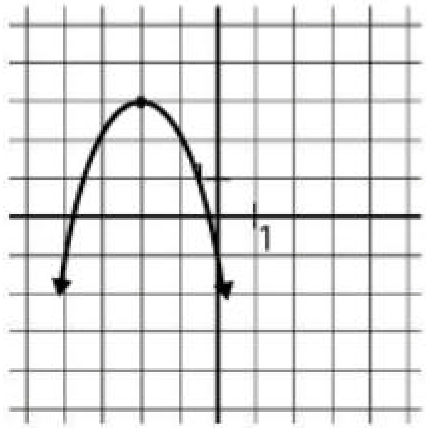
Once we have the x value, we substitute that into the function for x to calculate the y that goes with that x value. For $y = 3x^2 + 18x + 4$, *the x value of* *the vertex is* $x = \frac{-b}{2a} = \frac{-18}{2(3)} = -3$. *Plug that into* $y = 3x^2 + 18x + 4$ *for x and get* $y = 3(-3)^2 + 18(-3) + 4 = 27 - 54 + 4 = -23$. *The vertex is the ordered pair (-3, -23). I do not have room to graph it here.*

5. **Orientation:** Use your calculator to quickly graph the following functions. Try to plot the vertices and *y*-intercepts accurately.

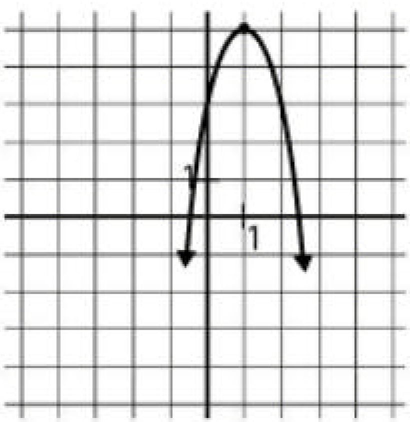
a.) $y = -x^2 - 4x - 4$



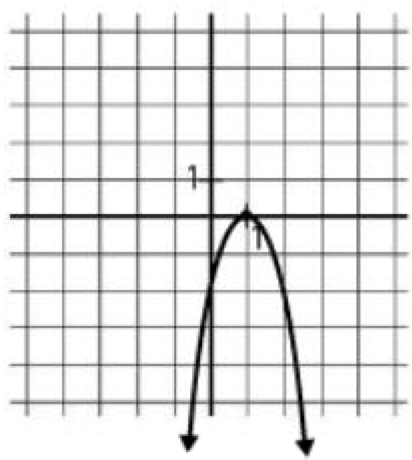
$y = -x^2 - 4x - 1$



b.) $y = -2x^2 + 4x + 3$

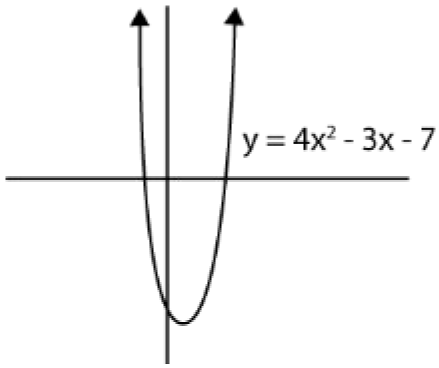


$y = -2x^2 + 4x - 2$



Notice all of these functions graphed as upside down parabolas (they opened downward) whereas the first set all opened upward. What do you think made the difference? Let's test your theory. Make up a quadratic function you think should open downward. Use your theory to guess its orientation. Then graph the function. Copy the graph and label it. Is the orientation what you thought it would be?

If the x^2 *term is negative, then the parabola opens down, like in the above four graphs. If the* x^2 *term is positive, the parabola opens up, like the four functions in* *question 1. To test this, I made up the function* $y = 4x^2 - 3x - 7$ *. It should open up since the coefficient 4 is positive. Below is its graph.*



6. **Summary:** The y-intercept of a quadratic function $y = ax^2 + bx + c$ is $y = c$.

The x value of the vertex of a quadratic function is $x = \frac{-b}{2a}$. To get the y value of the vertex, you simply substitute this value into the function for x and solve for y.

The orientation of a quadratic function is determined solely by the coefficient a. If this coefficient is positive, the parabola opens up. If this coefficient is negative, the parabola opens down.

There is nothing to be done here. These are just the main points of this worksheet.