Banking Application   NAME:
Savings account with multiple withdrawals Solutions

Here we will investigate finding the balance and interest earned on savings accounts with multiple withdrawals. Consider the problem with solution below.

On February 8, Mary deposits $5,000 into a savings account that pays 3½% interest compounded daily. On March 15, she withdraws $450. On April 20, she withdraws $1,000. What is the balance on May 1? How much interest did she earn through May 1?

Solution: I see four important dates: the day of the initial deposit (Feb. 8), the day of the first withdrawal (March 15), the day of the second withdrawal (April 20), and the end date (May 1).

You can think of this money as three separate amounts and how long they each was in the account. Now, of the original $5,000, only $3,550 of it (that is, $5,000 – $1,000 – $450) was in the bank from Feb. 8 – May 1. The first withdrawal of $450 was in the bank from Feb. 8 – March 15. The second withdrawal of $1,000 was in the bank from Feb. 8 – April 20.

Notice the periods of interest are Feb. 8 – March 15, Feb. 8 – April 20, and Feb. 8 – May 1. Notice each of these periods start on Feb. 8 and end on a withdrawal date or the end date.

We will find the compound amounts for these periods (that is, the original amounts plus the interest earned for each period) and add them. Her balance, then, will be this sum minus her withdrawals. The interest earned will be found by finding the interest earned for each of the three amounts and adding them.

To find the compound amounts and how much interest she earned during these three periods, we need to know how many days are in each period. I use the table "The Number of Each of the Days of the Year" in the back of the book.

The first period we are interested in is Feb. 8 – May 1. Since Feb. 8 is day 39 and May 1 is day 121, we subtract to see that this period has 82 days.

The second period we are interested in is Feb. 8 – March 15. Again, since Feb. 8 is day 39 and March 15 is day 74, we subtract to see that this period has 35 days.

The third period we are interested in is Feb. 8 – April 20. Again, since Feb. 8 is day 39 and April 20 is day 110, we subtract to see that this period has 71 days.

The following table organizes this information.

Amount in account	Dates	Number of days
$3,550	Feb. 8 – May 1	82
$450	Feb. 8 – March 15	35
$1,000	Feb. 8 – April 20	71

So, we need to figure the compound amounts and how much interest she earned during these three periods. This depends on the amount in the account and the number of days. We will use the table "Values of $(1 + i)^n$ for 3½ % Compounded Daily" provided on the last page of this worksheet. By looking up the number of days in the table for each period, I found the multiplier needed for our calculations. I added them to the table below. Make sure you can find these values yourself on the table.

Amount in account	Dates	Number of days	Multiplier from Interest table
$3,550	Feb. 8 – May 1	82	1.007893628
$450	Feb. 8 – March 15	35	1.003361641
$1,000	Feb. 8 – April 20	71	1.006831119

To find the compound amounts for each period, we multiply these values by the amounts in the account. To find the interest earned during each period, we subtract the original amounts from these compound amounts. Use the entire decimal number from the table. I show the calculations below. Column 3 shows the compound amount, which is how much is in the account with interest included. Column 4 shows the interest alone.

Amount in account	Multiplier from Interest table	Calculation for the compound amount	Interest earned
$3,550	1.007893628	$3,550 ∗ 1.007893628 = $3,578.02	$3,578.02 – $3,550 = $28.02
$450	1.003361641	$450 ∗ 1.003361641 = $451.51	$451.51 – $450 = $1.51
$1,000	1.006831119	$1,000 ∗ 1.006831119 = $1,006.83	$1,006.83 – $1,000 = $6.83

Now we are ready to find the amount in the account on May 1. This should be the sum of the compound amounts from column 3 above minus the two withdrawal amounts or $3,578.02 + $451.51 + $1,006.83 – $450 – $1,000 = $3,586.36. The book would do this last calculation slightly differently, using the interest earned along with the initial deposit. Notice their calculation, $5,000 + $28.02 + $1.51 + $6.83 – $450 – $1,000 would yield the same result. Do you see why?

Since column 4 above gives the interest earned for each of the three amounts in question, we simply add them to find the total interest this account earned. That is, $28.02 + $1.51 + $6.83 which is $36.36.

Try a couple on your own. Use the table on the next page as opposed to the one in the book to find the appropriate multiplier.

1. Sam and Bob have a joint savings account that pays 3½ % interest compounded daily. They deposit $14,000 on January 15. On March 2, Sam withdraws $2,500. On April 4, Bob withdraws $1,200. How much interest has the account earned through May 15? How much money is in the account on May 15?

Think of this as three deposits: a) $10,300 (which is $14,000 - $2,500 - $1,200) from Jan. 15 to May 15, b) $2,500 from Jan. 15 to Mar. 2, and c) $1,200 from Jan. 15 to April 4.

We use the "The Number of Each of the Days of the Year" table to find the number of days for each of the three periods. We find that $10,300 was in the account for 120 days, $2,500 was in the account for 46 days, and $1,200 was in the account for 79 days. We look these "number of days" up in the table on the last page of this worksheet and multiply each with their deposit amounts. This finds us the compound amounts. Round each to two decimal places and add them to get the total.

> *10,300 * 1.011572750 = $10,419.20*
>
> *2,500 * 1.004420489 = $2,511.05*
>
> *1,200 * 1.007603742 = $1,209.12*

*Add these together to get $14,139.37. (By the way, since the book rounds each of these to two decimal places before adding, you may get a slight rounding error if you try to put the whole thing into your calculator as 10,300 * 1.011572750 + 2,500 * 1.004420489 + 1,200 * 1.007603742. This is annoying to me, because I prefer putting the whole thing in at once. The book and MML disagrees. And, unfortunately, I think MML would count it wrong even if you were only off by a few cents.)*

Then to get the balance on May 15, we need to subtract the withdrawals from that total. I do this below.

Balance on May 15: 14,139.37 – 2,500 – 1,200 = $10,439.37 Balance

To find the interest earned, it is probably easiest to find each compound amount above separately (rather than add them at the same step) and subtract the principal from each. I do this below.

*10,300 * 1.011572750 = $10,419.20 So interest earned is $10,419.20 – $10,300 or $119.20.*

*2,500 * 1. 004420489 = $2,511.05 So interest earned is $2,511.05 – $2,500 or $11.05.*

*1,200 * 1.007603742 = $1,209.12 So interest earned is $1,209.12 – $1,200 or $9.12.*

Add these three interest values to find the total interest earned is $139.37.

2. Barb opens a savings account on April 1 by depositing $1,200. It pays her 3½ % interest compounded daily. She withdraws $200 on May 5 and another $450 on May 30. She wants to buy a sofa costing $600 on July 1. Does she have enough money on July 1 to do that? How much more does she need or how much extra money has she got?

Think of this as three deposits: a) $550 (which is $1,200 - $200 - $450) from April 1 to July 1, b) $200 from April 1 to May 5, and c) $450 from April 1 to May 30.

We use the "The Number of Each of the Days of the Year" table to find the number of days for each of the three periods. We find that $550 was in the account for 91 days, $200 was in the account for 34 days, and $450 was in the account for 59 days. We look these "number of days" up in the table on the last page of this worksheet and multiply each with their deposit amounts. This finds us the compound amounts.

$$550 * 1.008763788 = \$554.82$$

$$200 * 1.003265438 = \$200.65$$

$$450 * 1.005673296 = \$452.55$$

Add these to get the total of $1,208.02. Then to get the balance on July 1, we need to subtract the withdrawals from that total. I do this below.

Balance on May 15: 1208.02 – 200 – 450 = $558.02 Balance

So her account will have $558.02 on July 1 but she needs $600 to buy her sofa. So she does not

have enough money and will need $41.98 more ($600 - $558.02).

	Values of $(1+i)^n$ for 3½ % Compounded Daily						
Number of Days	Compound Amount Multiplier	Number of Days	Compound Amount Multiplier	Number of Days	Compound Amount Multiplier	Number of Days	Compound Amount Multiplier
1	1.000095890	41	1.003939056	81	1.007796990	121	1.011669750
2	1.000191790	42	1.004035324	82	1.007893628	122	1.011766759
3	1.000287699	43	1.004131602	83	1.007990276	123	1.011863778
4	1.000383617	44	1.004227888	84	1.008086932	124	1.011960806
5	1.000479544	45	1.004324184	85	1.008183598	125	1.012057843
6	1.000575480	46	1.004420489	86	1.008280273	126	1.012154890
7	1.000671426	47	1.004516803	87	1.008376958	127	1.012251946
8	1.000767381	48	1.004613127	88	1.008473651	128	1.012349011
9	1.000863345	49	1.004709460	89	1.008570354	129	1.012446086
10	1.000959318	50	1.004805802	90	1.008667067	130	1.012543170
11	1.001055300	51	1.004902153	91	1.008763788	131	1.012640263
12	1.001151292	52	1.004998513	92	1.008860519	132	1.012737365
13	1.001247293	53	1.005094883	93	1.008957259	133	1.012834477
14	1.001343303	54	1.005191262	94	1.009054008	134	1.012931598
15	1.001439322	55	1.005287650	95	1.009150767	135	1.013028729
16	1.001535350	56	1.005384048	96	1.009247535	136	1.013125868
17	1.001631388	57	1.005480454	97	1.009344312	137	1.013223017
18	1.001727435	58	1.005576870	98	1.009441098	138	1.013320176
19	1.001823491	59	1.005673296	99	1.009537894	139	1.013417344
20	1.001919556	60	1.005769730	100	1.009634699	140	1.013514521
21	1.002015631	61	1.005866174	101	1.009731513	141	1.013611707
22	1.002111714	62	1.005962627	102	1.009828337	142	1.013708903
23	1.002207807	63	1.006059089	103	1.009925170	143	1.013806107
24	1.002303909	64	1.006155560	104	1.010022012	144	1.013903322
25	1.002400021	65	1.006252041	105	1.010118863	145	1.014000545
26	1.002496141	66	1.006348531	106	1.010215724	146	1.014097778
27	1.002592271	67	1.006445030	107	1.010312594	147	1.014195021
28	1.002688410	68	1.006541538	108	1.010409473	148	1.014292272
29	1.002784558	69	1.006638056	109	1.010506362	149	1.014389533
30	1.002880716	70	1.006734583	110	1.010603260	150	1.014486803
31	1.002976882	71	1.006831119	111	1.010700167	151	1.014584083
32	1.003073058	72	1.006927665	112	1.010797083	152	1.014681372
33	1.003169243	73	1.007024219	113	1.010894009	153	1.014778670
34	1.003265438	74	1.007120783	114	1.010990944	154	1.014875977
35	1.003361641	75	1.007217357	115	1.011087889	155	1.014973294
36	1.003457854	76	1.007313939	116	1.011184842	156	1.015070621
37	1.003554076	77	1.007410531	117	1.011281805	157	1.015167956
38	1.003650307	78	1.007507132	118	1.011378777	158	1.015265301
39	1.003746548	79	1.007603742	119	1.011475759	159	1.015362655
40	1.003842797	80	1.007700362	120	1.011572750	160	1.015460019

Values for $(1+i)^n$ were calculated using n as the number of days and $i = .035/365$