Math 128
NAME:
Inverses of Trigonometric Functions Cosine, Sine, and Tangent

1. Reflect the graph of $y=\sin x$ about the line $y=x$. You'll recall this creates the inverse of the function $y=\sin x$, which "undoes" $y=\sin x$. We denote this inverse relationship by $y=\sin ^{-1} x$.

2. You'll notice that the reflection is not a function. It has more than one $y$ value for many of the $x$ values. How could we restrict the domain of $y=\sin x$ so that its inverse was a function? There are many correct answers here. Select an interval of $x$ values for the original function so that its inverse is a function.
3. The usual domain restriction for $\mathrm{y}=\sin \mathrm{x}$ is $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. Concentrate only on this portion of the graphs to complete this table.

| $\sin ^{-1}(.7)=$ | $\sin (\pi / 4)=$ |
| :--- | :--- |
| $\sin ^{-1}(0)=$ | $\sin (0)=$ |
| $\sin ^{-1}(1)=$ | $\sin (\pi / 2)=$ |

4. Reflect the graph of $y=\cos x$ about the line $y=x$. This creates the inverse of the function $y=\cos x$, denoted by $y=\cos ^{-1} x$. Just as before the inverse is not a function unless the domain of $y=\cos x$ is restricted. We usually restrict the domain of $y=\cos x$ to $0 \leq x \leq \pi$. Cross out the remainder of the graph and only reflect the restricted domain part.

5. The usual domain restriction for $\mathrm{y}=\cos \mathrm{x}$ is $0 \leq x \leq \pi$. We will only need this portion of the graphs to complete this table.

| $\cos ^{-1}(.7)=$ | $\cos (\pi / 4)=$ |
| :--- | :--- |
| $\cos ^{-1}(0)=$ | $\cos (\pi / 2)=$ |
| $\cos ^{-1}(-.5)=$ | $\cos (2 \pi / 3)=$ |

6. Notice if $f(x)=\cos x$ and $g(x)=\cos ^{-1} x$ are inverses, then $f(g(x))=x$ or rather, $\cos \left(\cos ^{-1}(x)\right)=x$ or alternatively $\cos ^{-1}(\cos (x))=x$. Similar things can be said of $y=\sin x$ and $y=\sin ^{-1} x$. Use the results of numbers three and five above to verify these equations.
7. Consider this graph of $\mathrm{y}=\tan \mathrm{x}$ with the restricted domain of $-\frac{\pi}{2}<x<\frac{\pi}{2}$. (Notice the values $-\pi / 2$ and $\pi / 2$ are not included as the function is undefined at these values.) Reflect it over the $y=x$ line. Only reflect that part in the restricted domain.

8. Using the wording of asymptotes, describe the graph of $y=\tan ^{-1} x$.
9. Use your graphs with the appropriate restricted domains to fill in the following table.

| Original <br> Trig Fnc | Original <br> Trig Fnc's <br> Domain | Original <br> Trig Fnc's <br> Range | Inverse | Inverse's <br> Domain | Inverse's <br> Range |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{y}=\sin \mathrm{x}$ | $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ |  | $\mathrm{y}=\sin ^{-1} \mathrm{x}$ |  |  |
| $\mathrm{y}=\cos \mathrm{x}$ | $0 \leq x \leq \pi$ |  | $\mathrm{y}=\cos ^{-1} \mathrm{x}$ |  |  |
| $\mathrm{y}=\tan \mathrm{x}$ | $-\frac{\pi}{2}<x<\frac{\pi}{2}$ |  | $\mathrm{y}=\tan ^{-1} \mathrm{x}$ |  |  |

10. What is true of the relationship between the domain and range of a trig function and that of its inverse?
