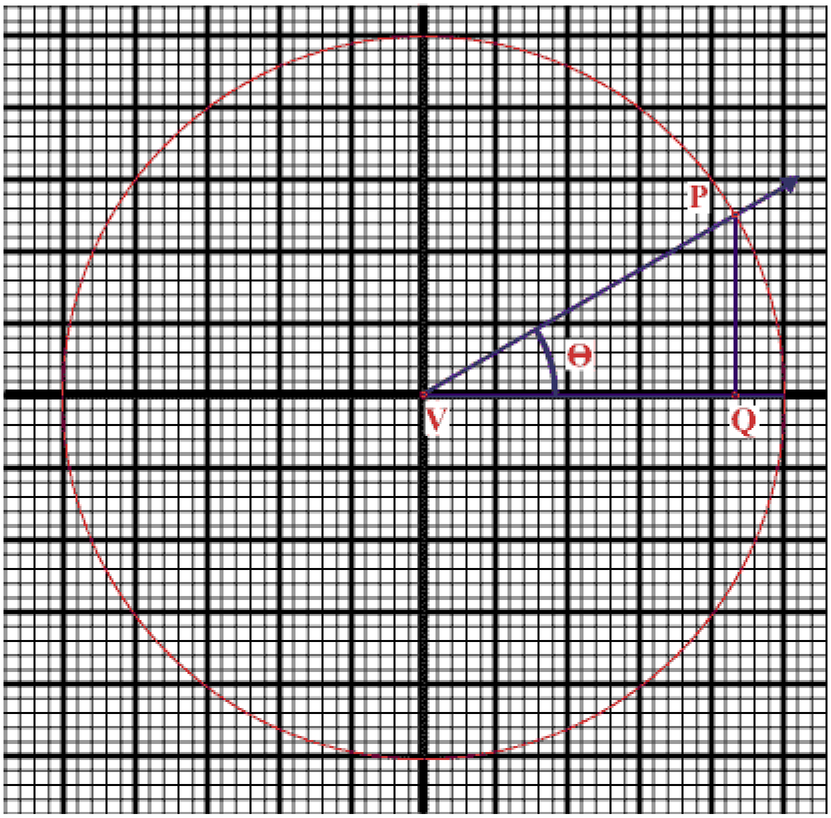
Math 128 Trigonometry **NAME:**
SOH CAH TOA/ unit circles/ 30,60,90 triangles/

SOH CAH TOA:

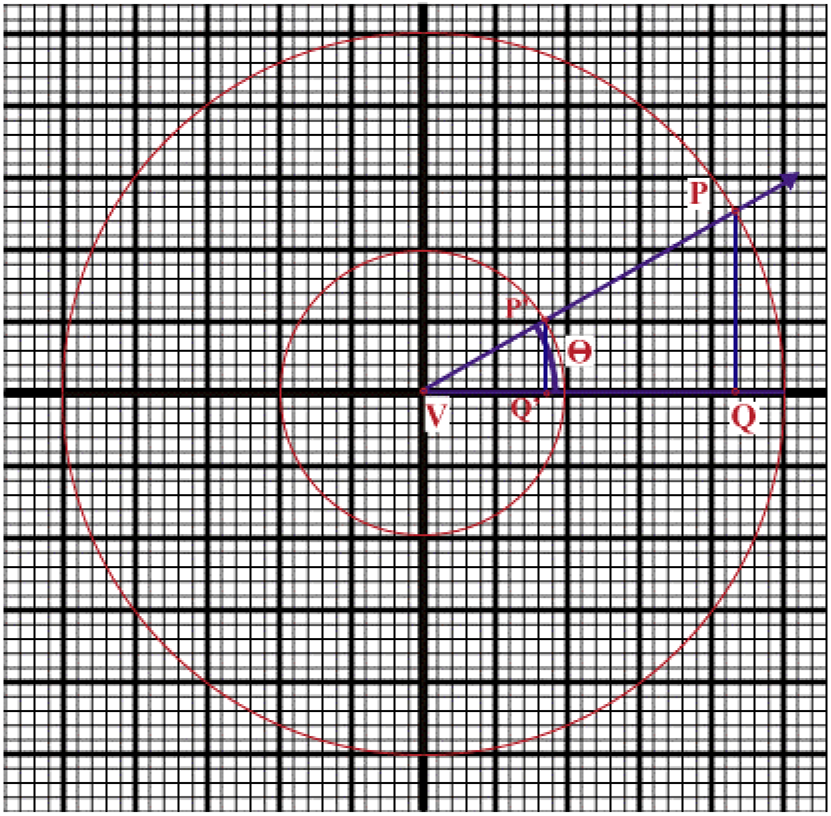
1. Consider the following picture. **On the drawing, ten boxes represents one unit.** Notice angle è is formed by the positive x-axis and the ray through (0,0) and the point P. The point Q is directly below P on the x-axis. The vertex of the angle è is labeled V.



What are the coordinates of points P and Q? **Remember, ten boxes represents one unit.** Label the lengths of the triangle's sides accordingly. You can use the Pythagorean Theorem to find the length of the hypotenuse.

Using SOH CAH TOA, find the sine, cosine and tangent of the angle è.

2. Consider the revised drawing below. Notice the new circle has a radius of 1 unit. Also, the new points P' and Q', which correspond to P and Q, are pictured. Determine the points' coordinates and calculate the lengths of the new triangle's sides.



Using this new triangle and SOH CAH TOA, find the sine, cosine and tangent of the angle è.

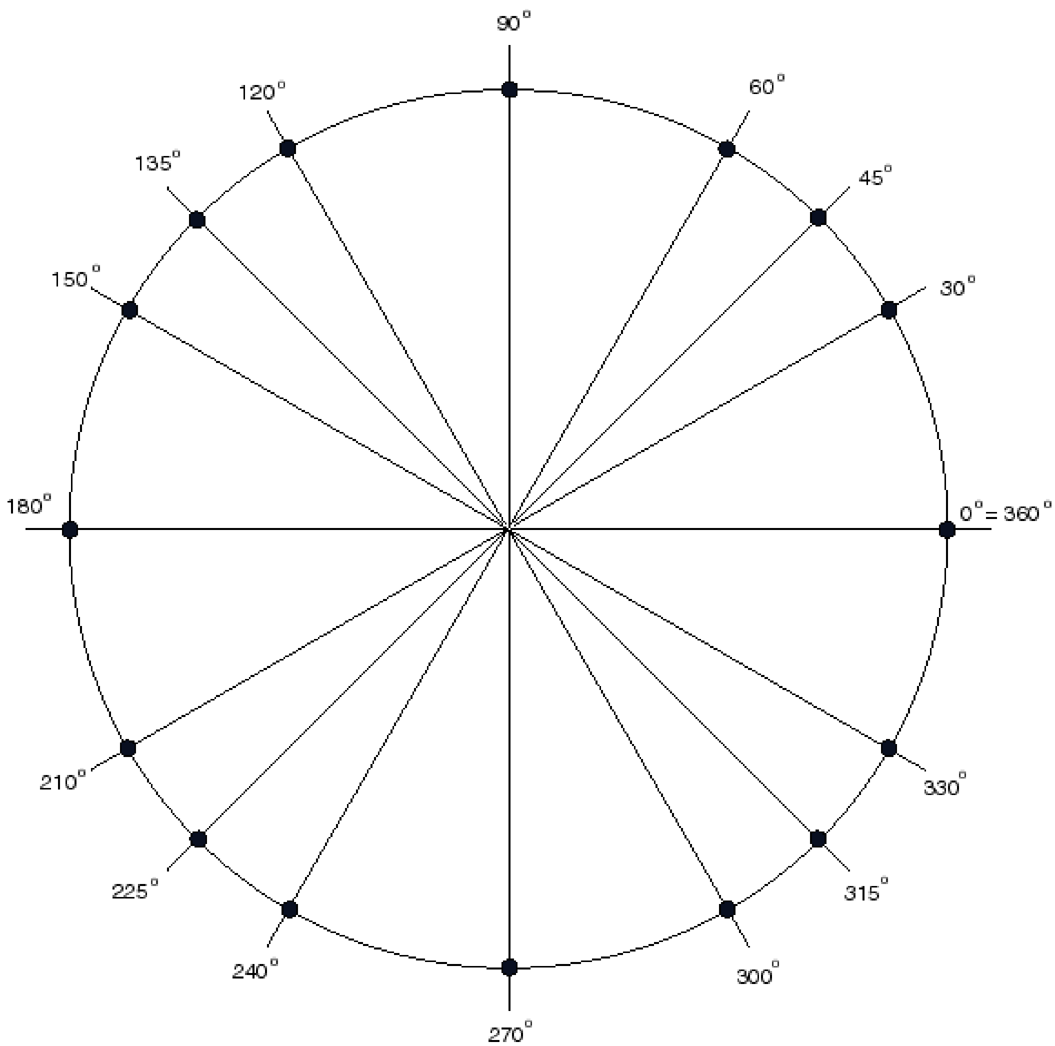
3. Any conclusions?

Unit Circle Approach:

4. Consider the circle and the angle measurements below. This is what is called a unit circle and it's used to determine the sine, cosine, and the other trigonometric functions of these reference angles.

Knowing the radius of a unit circle is one unit, fill in the coordinates of each point. The first five are done for you. Symmetry of the angles and the first five points' coordinates will tell you the other coordinates.
Remember SOH CAH TOA whispers through the trees.

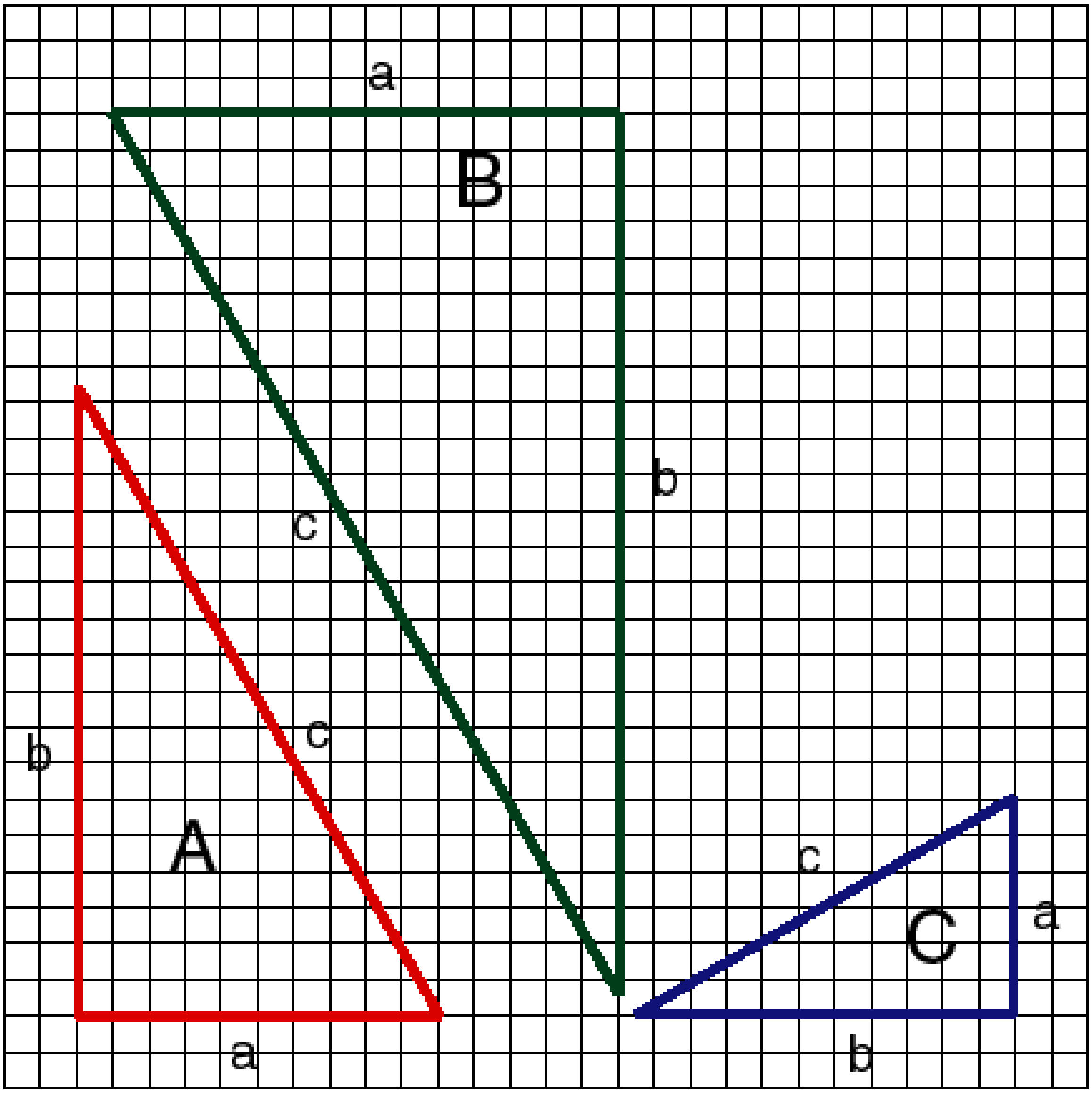
Complete the table below detailing the sines, cosines, and tangents of these angles. (Use the signs of the points' coordinates. For example, the work for è = 210 º is done for you. Notice how the signs of the coordinates are noted and how they are used to find the trigonometric functions.)



è in degrees	è in radians	Point's coordinates (a, b)	Sine of è	Cosine of è	Tangent of è
0	$0 * \left(p/180\right) = 0p/6 = 0$	(1,0)			
30	$30 * \left(p/180\right) = p/6 \approx .52$	$\left(\sqrt{3}/2, \frac{1}{2}\right)$			
45	$45 * \left(p/180\right) = p/4 \approx .79$	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$			
60	$60 * \left(p/180\right) = p/3 \approx 1.05$	$\left(\frac{1}{2}, \sqrt{3}/2\right)$			
90	$90 * \left(p/180\right) = p/2 \approx 1.57$	$(0,1)$			
120					
135					
150					
180		(-1,0)			
210	$210 * \left(p/180\right) = 7p/6 \approx 3.66$	$\left(-\sqrt{3}/2, \frac{-1}{2}\right)$	$\sin q = b = \frac{-1}{2}$	$\cos q = a = -\sqrt{3}/2$	$\tan q = b/a = 1/\sqrt{3}$
225					
240					
270		(0,-1)			
300					
315					
330					

30º/60º/90º triangles:

5. Using the grid, measure the lengths of the sides of these 30º/60º/90º triangles (in terms of number of units (boxes)). To find the length of the hypotenuse, measure it in inches, then multiply by 5 to get its length in terms of number of units (boxes). There is a table on the next page to organize the information.



Triangle	Side a (opposite 30º)	Side b (opposite 60º)	Side c (opposite 90º)
A			
B			
C			

What do you notice about the relationships among the three sides' lengths?

6. Use the table from number 5 and SOH CAH TOA to find the following.

a. Find the sine of 30º.

b. Find the cosine of 60º.

c. Find the tangent of 30º.

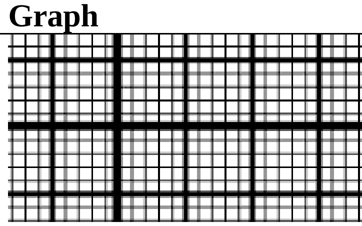
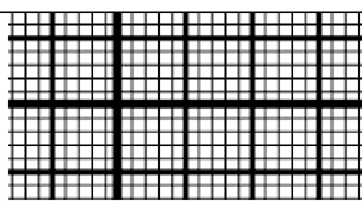
7. Why would this not work to find the tangent of 90º?

graphs of trigonometric functions

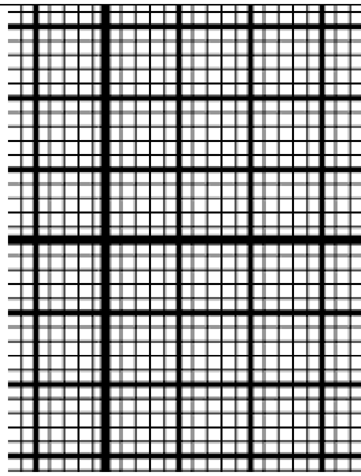
8. Use the unit circle table to find the cosecant, secant, and cotangent of the angles in the table below.

è (in radians)	**Cosecant of è**	**Secant of è**	**Cotangent of è**
π/4			
2π/3			
π			
2π			

9. Draw rough sketches of the graphs of the trigonometric functions in the table below. You may use your calculator but also verify for yourself the numbers in your table could also be plotted. Use a window of [-ë, 3 ë] x [-1.25, 1.25] for the sine and cosine graphs. Use a window of [-ë, 3 ë] x [-3, 3] for the other four graphs. Label an appropriate scale on your graphs. From your drawings, determine the domain and range of the six functions.

Function	**Graph**	**Domain**	**Range**
f(è) = sin è			
f(è) = cos è			

f(è) = tan è			
f(è) = csc è			
f(è) = sec è			

f(è) = cot è			

Notice in the graph of f(è) = cot è, there are vertical asymptotes on every multiple of ë. Use the graph of f(è) = tan è and the relationship between cot è and tan è to explain why this is the case.

basic identities/ reference angle theorems

10. Use your calculator to find the pairs of values listed below. Make sure your calculator is set in radians.

a.) sin (ë/6)

sin (ë/6 + 2 ë)

b.) sin(ë/2)

sin(ë/2 + 4 ë)

c.) sin(2.36)

sin(2.36 + 6 ë)

Answer the following questions about number 10.

a.) You should have noticed that each pair’s values were equal. Look at the graph of f(è) = sin è to explain why this is true.

b.) In fact, verify that sin è = sin(è +2 ëk) for any integer k. Use your calculator’s graphing capabilities if you want.

c.) Look at the graph of f(è) = cos è and derive a similar formula.

d.) Look at the graphs of f(è) = csc è, f(è) = sec è, f(è) = tan è, and f(è) = cot è to derive similar formulas.

11. Use the unit circle table to complete this table.

è in degrees	**è in radians**	**cos è**	**sin è**	**sin è/cos è**	**tan è**
210					
	$\frac{11}{6}p$				

Verify that tan è = sin è / cos è.

basic identities/ reference angle theorems

12. Open your book to page 426-428. There are several identities, trigonometric formulas that are true for any angle è. Note that $\sin^2$ è means (sin è)^2.

Choose one or two angles in the unit circle table to verify the following identities.

a.) $\sin^2$ è + $\cos^2$ è = 1

b.) cot è = cos è / sin è

c.) cos (-è) = cos è

Now choose three of the other identities. State the rule as it's stated in the book, give the value of è you will use, and then substitute è to show the identity is true.

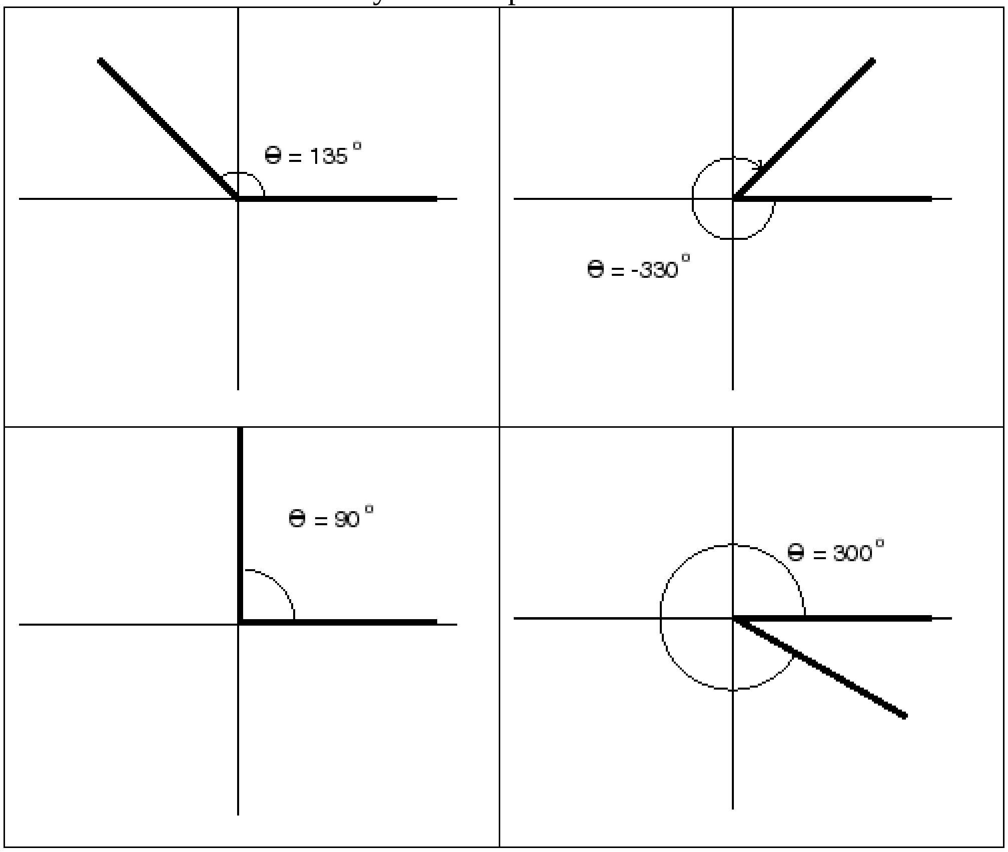
1.)

2.)

3.)

reference angle theorems

13. For each of the following angles è drawn, draw in and label the reference angle á. Then select one of the identities on page 435 and verify the equation using the unit circle table. Use a different identity for each picture.



14. Consider questions 1 and 2 of this worksheet. Using the fact that you know sin è, cos è, and tan è, what is è?