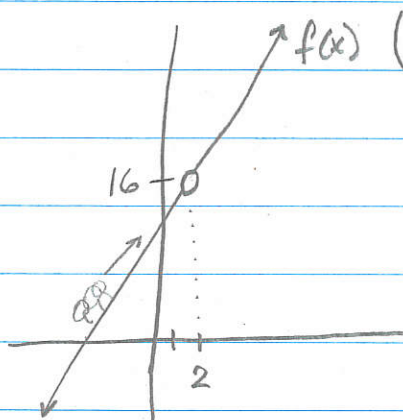


1.1: Limits: A Numerical + Graphical Approach

(A) Consider $f(x) = \frac{4x^2 - 16}{x - 2}$ $[-5, 10] \times [-5, 25]$



Notice $f(x)$ is undefined at $x=2$.

What does y approach as x approaches 2 from the left?

Table of Values

Indpt var: x
Dependent var: y

from left

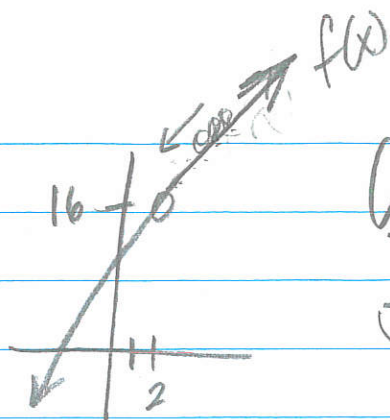
x	y
1.9	15.6
1.99	15.96
1.999	15.996

what do these values appear to be approaching? (16)

$$\lim_{x \rightarrow 2^-} f(x) = 16$$

The limit, as x approaches 2 from the left, of $f(x)$ is 16.

(This is a left-sided limit).



What is y approaching
as x approaches 2 from
the right?

$$\lim_{x \rightarrow 2^+} f(x) = 16 \quad \left(\begin{array}{l} \text{Right-} \\ \text{sided} \\ \text{limit} \end{array} \right)$$

x	y
2.1	16.4
2.01	16.04
2.001	16.004

It definitely looks like
 $y \rightarrow 16$ as $x \rightarrow 2^+$.

Handout: Thm 1: Since $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$

and both are 16, we
can say $\lim_{x \rightarrow 2} f(x) = 16$.

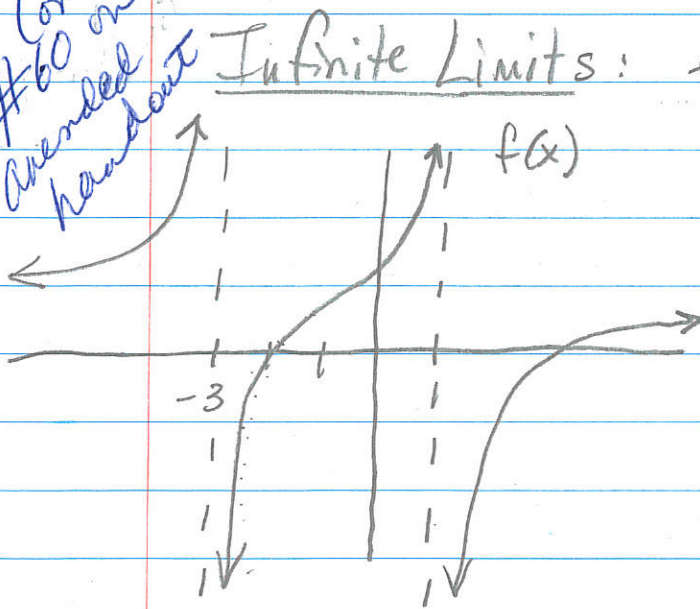
(B) Expls: Piecewise fnc: A fnc whose rule
depends on the piece of the
domain you're in.

handout

#21
#29
#30

(c) Limits involving infinity: (exploring either $x \rightarrow \pm \infty$ or "we'll see" $y \rightarrow \pm \infty$.)

(online)
#60 on
amended
handout



Infinite Limits: expl (#54) $\lim_{x \rightarrow -3} f(x)$

→ We need to find $\lim_{x \rightarrow -3^-} f(x)$ and $\lim_{x \rightarrow -3^+} f(x)$.

So, as we approach -3 from the left, y approaches ∞
 $\rightarrow \lim_{x \rightarrow -3^-} f(x) = \infty$

Technically, this limit is said to not exist (as it's not a real number). But we'll use this handy notation.

And, as we approach -3 from the right, y approaches $-\infty$.
 $\rightarrow \lim_{x \rightarrow -3^+} f(x) = -\infty$

So, what's $\lim_{x \rightarrow -3} f(x)$?

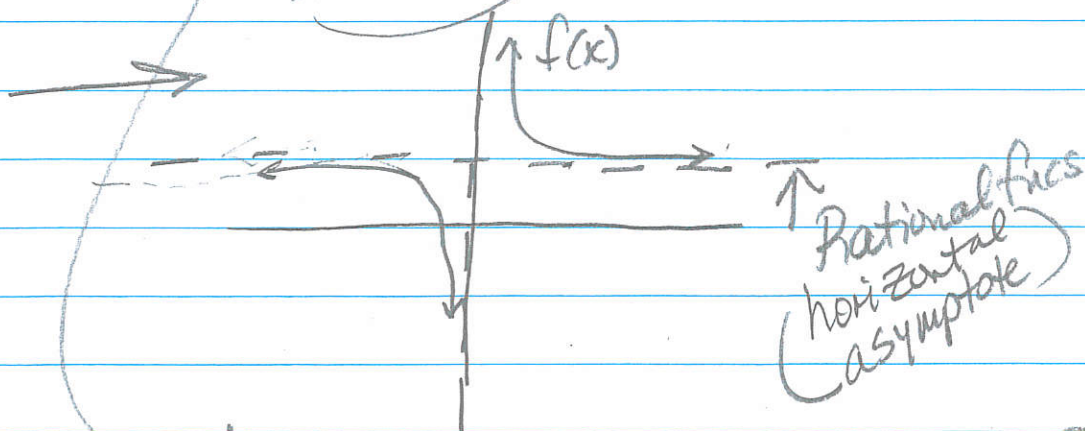
It does not exist because left and right-hand limits are different.

Limits at Infinity:

#68 Graph the func and then find the limit. If the limit does, say so.

$$f(x) = \frac{1+3x}{1-x} \quad \text{Standard window}$$

find $\lim_{x \rightarrow \infty} f(x)$



What is y approaching as $x \rightarrow \infty$?
(looks like 3)

$$\lim_{x \rightarrow \infty} f(x) = 3$$

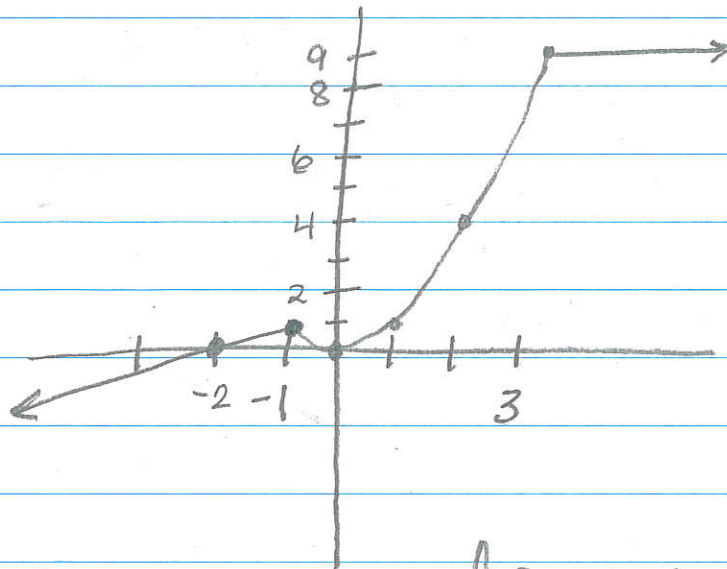
④ Other examples:

Graphing piecewise fns: #78

Graph and find limit. If dne, say so.

$$G(x) = \begin{cases} 2+x & \text{for } x \leq -1 \\ x^2 & \text{for } -1 < x < 3 \\ 9 & \text{for } x \geq 3 \end{cases}$$

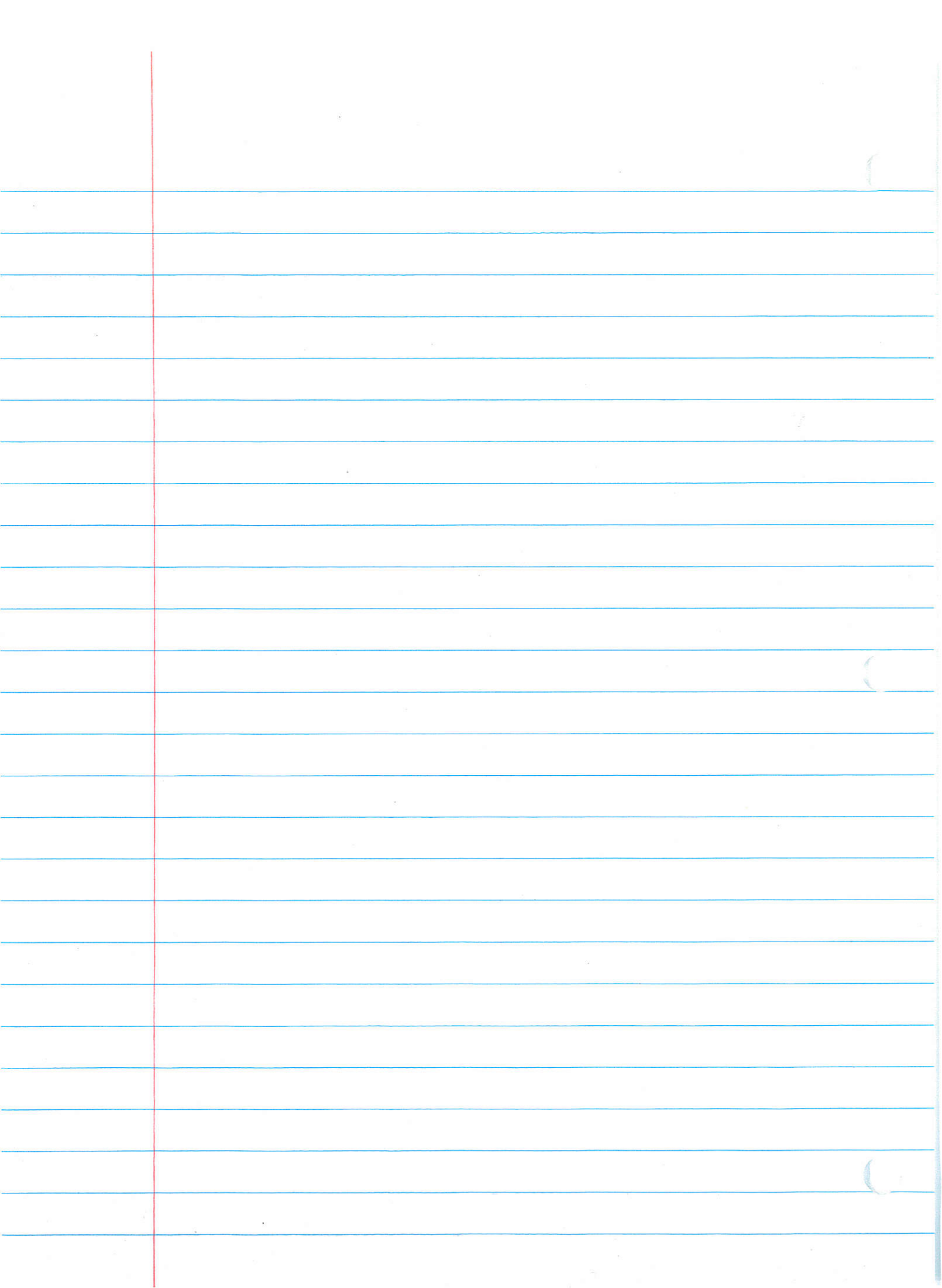
Find $\lim_{x \rightarrow -1} G(x)$ and $\lim_{x \rightarrow 3} G(x)$.



$$\lim_{x \rightarrow -1} G(x) = \textcircled{1}$$

$$\lim_{x \rightarrow 3} G(x) = \textcircled{9}$$





Theorem 1

For a function $f(x)$ and L a real number,

$$\lim_{x \rightarrow a} f(x) = L \text{ if and only if } \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L.$$



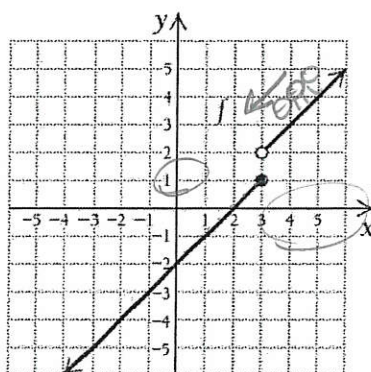
The $\lim_{x \rightarrow a} f(x)$ does not exist if the left-hand or right-hand limit does not exist or if the left-hand or right-hand limit exist but are different.

The existence of $\lim_{x \rightarrow a} f(x)$ does not require that $f(a)$ exists. If $f(a)$ does exist, it does not automatically follow that $\lim_{x \rightarrow a} f(x) = f(a)$. (See Example 5.)

1.1 Examples done in class:

For Exercises 21 and 22, consider the function f given by

$$f(x) = \begin{cases} x - 2, & \text{for } x \leq 3, \\ x - 1, & \text{for } x > 3. \end{cases}$$



If a limit does not exist, state that fact.

21. Find

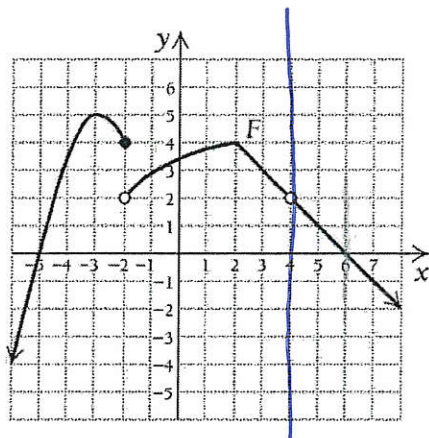
a. $\lim_{x \rightarrow 3^-} f(x) = 1$

b. $\lim_{x \rightarrow 3^+} f(x) = 2$

c. $\lim_{x \rightarrow 3} f(x)$ d.n.e.

"does not exist"

For Exercises 25–34, use the following graph of F to find each limit. When necessary, state that the limit does not exist.



$$\lim_{x \rightarrow 4^-} F(x) = 2$$

$$\lim_{x \rightarrow 4^+} F(x) = 2$$

they match.

"Wall Method"

29. $\lim_{x \rightarrow 4} F(x) = 2$

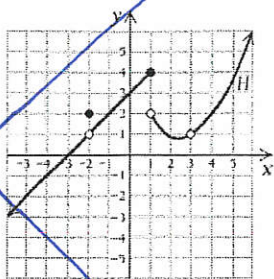
30. $\lim_{x \rightarrow 6} F(x) = 0$

$F(4)$ d.n.e.

$F(6) = 0$ (same value as

$\lim_{x \rightarrow 6} F(x)$)

For Exercises 45–56, use the following graph of H to find each limit. When necessary, state that the limit does not exist.



Wrong one

52. $\lim_{x \rightarrow 1} H(x)$