

The German mathematician Gottfried von Leibniz developed this notation as he worked to invent calculus in the 1670's.

Derivative Notation:

In the last section, we established that the **derivative of f** , denoted $f'(x)$, is the function

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ provided this limit exists. We often say $y = f(x)$ and so sometimes we write this derivative as y' (pronounced "y-prime").

Gottfried von Leibniz used the notation $\frac{dy}{dx}$ to mean the "derivative of y with respect to x ". We pronounce it as simply " $dy dx$ ".

It is important to note that this $\frac{dy}{dx}$ should not be thought of as a fraction with a common factor of d on top and bottom. The symbol should be taken as a whole and thought of as the derivative of the function $y = f(x)$ or rather $f'(x)$.

Okay, so we're to the important bit.
We may use $\frac{dy}{dx}$ to mean $f'(x)$. In certain problems, this is very helpful.

You might also see notation such as $\frac{df}{dx}$, $\frac{d}{dx} f(x)$, $D_x(f(x))$, $y'(x)$, ...

$$3 + 2 = 5$$

In particular, the notation $\frac{d}{dx} f(x)$ is used often in our book. We read this as " $d dx$ of f of x ".

Again, it is super important to see this not as a fraction but as what is called an **operator**. (The familiar plus and minus signs are also **operators** which are used to tell us to do certain things to numbers.) This notation means that we apply the derivative function to the function $f(x)$. In other words, we find $f'(x)$.

To show that we are evaluating a derivative at a specific x -value a , we might write $\frac{dy}{dx} \Big|_{x=a}$ or $y'(a)$, among others. Most common in this class will be good old $f'(a)$.

Finding Derivatives (Shortcut Rules):

You'll probably be pretty happy to learn that we are *not* going to continue using the difference quotient to find the derivatives of functions. Over the next few sections, we will be given shortcut rules that we will use unless asked specifically to use the difference quotient. Here are the first rules.

The book does provide proofs for many of these rules but we will skip those for now. Here are the rules we will work with in this section. (I copied these from another book so the theorem numbers do *not* match our book.)

THEOREM 3.2 Constant Rule

If c is a real number, then $\frac{d}{dx}(c) = 0$.

$$\frac{d}{dx}(3) = 0$$

$y = 3$
rate of change =

THEOREM 3.3 Power Rule

If n is a nonnegative integer, then $\frac{d}{dx}(x^n) = nx^{n-1}$.

Our book actually says that this rule is true for $n \in \mathbb{R}$.

$$f(x) = x^4$$

$$\frac{d}{dx}f(x) = \frac{d}{dx}x^4 = 4x^{4-1}$$

$$\frac{d}{dx}x^4 = 4x^3$$

THEOREM 3.4 Constant Multiple Rule

If f is differentiable at x and c is a constant, then

$$\frac{d}{dx}(cf(x)) = c f'(x).$$

$c \in \mathbb{R}$

THEOREM 3.5 Sum Rule

If f and g are differentiable at x , then

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x).$$

This, as you might imagine, works for subtraction too.

Let's use these bad boys!

This is really handy because it tells us to deal with big functions term by term.

Constant

expl 1: Use the rules to find the following derivatives. You can label your answers using prime notation (like $f'(x)$).

a.) $f(x) = \frac{5}{x} = 5x^{-1}$ ← need deriv of this

$f'(x) = 5(-1)x^{-2} = -5x^{-2}$
 $= \frac{-5}{x^2}$

Recall an alternative way to write $1/x = x^{-1}$

Power Rule
 $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

b.) $f(x) = 5x^3$

$f'(x) = 5 \cdot 3x^{3-1}$

$f'(x) = 15x^2$

c.) $f(t) = 5\sqrt{t} = 5t^{1/2}$

$f'(t) = 5(\frac{1}{2})t^{\frac{1}{2}-1} = \frac{5}{2}t^{-\frac{1}{2}}$

$f'(t) = \frac{5}{2}t^{-1/2}$

Recall an alternative way to write radicals.

$\sqrt[n]{t} = t^{1/n}$

d.) $f(x) = \frac{5}{3}x^3 + 6x^2 - \frac{2}{3}x + 7$

$f'(x) = \frac{5}{3} \cdot 3x^2 + 6 \cdot 2x^1 - \frac{2}{3} + 0$

$f'(x) = 5x^2 + 12x - \frac{2}{3}$

Power Rule

Constant Multiple Rule

Just go term by term.

Constant Rule

e.) $g(x) = -3x^{-5} - 4x^1$

$g'(x) = -3(-5)x^{-6} - 4 \cdot 1x^{1-1}$

$g'(x) = 15x^{-6} - 4 \cdot x^0 = 15x^{-6} - 4$

$\frac{d}{dx}(-\frac{2}{3}x^1) = -\frac{2}{3} \cdot 1 \cdot x^{1-1} = -\frac{2}{3}x^0$

expl 2: Use some alternative notations from page 1 to rename $g'(x)$ from part e above.

$g'(x) = 15x^{-6} - 4$

$y = g(x)$

$\frac{d}{dx}(g(x)) = \frac{15}{x^6} - 4$

or $\frac{dg}{dx}$ or y'

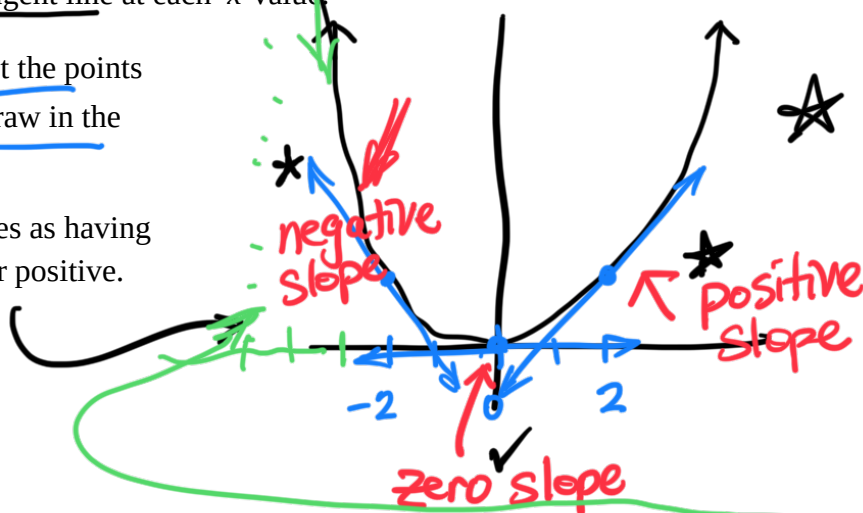
$x^0 = 1$
 for any non-zero number x

Derivative of a Function at a Specific x-value:

For the function $f(x) = x^2$, we know $f'(x) = 2x$. Also, remember that we learned that this derivative is the slope of the tangent line at each x-value.

(Draw a graph of $f(x) = x^2$. Plot the points for the x-values -2, 0, and 2. Draw in the tangent lines at these points.)

expl 3a: Label these tangent lines as having slopes that are negative, zero, or positive.



expl 3b: Let's calculate these slopes. They are the values $f'(-2)$, $f'(0)$, and $f'(2)$.

★ $f(x) = x^2$
 $f'(x) = 2x$

$f'(-2) = 2(-2) = -4$
 $f'(0) = 2 \cdot 0 = 0$ (hor. tangent line)
 $f'(2) = 2 \cdot 2 = 4$

expl 3c: Find the x-value such that its tangent line has a slope of -10.

$x = ?$

$f'(x) = 2x$

$-10 = 2x$

$\frac{-10}{2} = \frac{2x}{2}$

$x = -5$

$g(3) = 999$ ★

expl 4: Find the equation of the line tangent to the graph of $g(x) = 3x^2 + 4x^5$ at the point (3, 999).

Write your final answer in slope-intercept form

$g'(x) = 3 \cdot 2x + 4 \cdot 5x^4$

$g'(x) = 6x + 20x^4$

$g'(3) = 6 \cdot 3 + 20 \cdot 3^4$

$g'(3) = 1638$

derivative = slope tangent line

Want eqn of line with slope = $m = 1638$ and point (3, 999)

$y - y_1 = m(x - x_1)$

$y - 999 = 1638(x - 3)$

$y - 999 = 1638x - 4914$

$y = 1638x - 3915$

"dP/dt"

expl 5: The population of Kingsville has a population of $P(t)$ given by $P(t) = 100,000 + 2,000t^2$ where t is the number of years since the population was 100,000 people. Answer the following questions.

a.) Find the growth rate of the population, or $\frac{dP}{dt} = 0 + 2000 \cdot 2 \cdot t = \frac{dP}{dt} = 4000t$

b.) What is the population ten years after it reached 100,000?

$$P(t) = ? \quad t = 10 \rightarrow P(10) = 100,000 + 2000(10)^2 = 100,000 + 200,000$$

c.) Evaluate the growth rate at $t = 10$. Interpret.

$$\left. \frac{dP}{dt} \right|_{t=10} = 4000(10) = 40,000$$

At $t = 10$, the population is growing at the rate of 40,000 people per year.

$P(10) = 300,000 \rightarrow$ Pop is 300,000 people ten years after pop reached 100,000.

A Connection Between the Slope of a Tangent Line and Decreasing/Increasing Graphs:

The slope of a tangent line is the rate of change of the function at that point. If a graph is decreasing or increasing, what does that mean for this rate of change? Draw a nice, curvy polynomial function below and draw in several tangent lines with the following criteria. Label each.

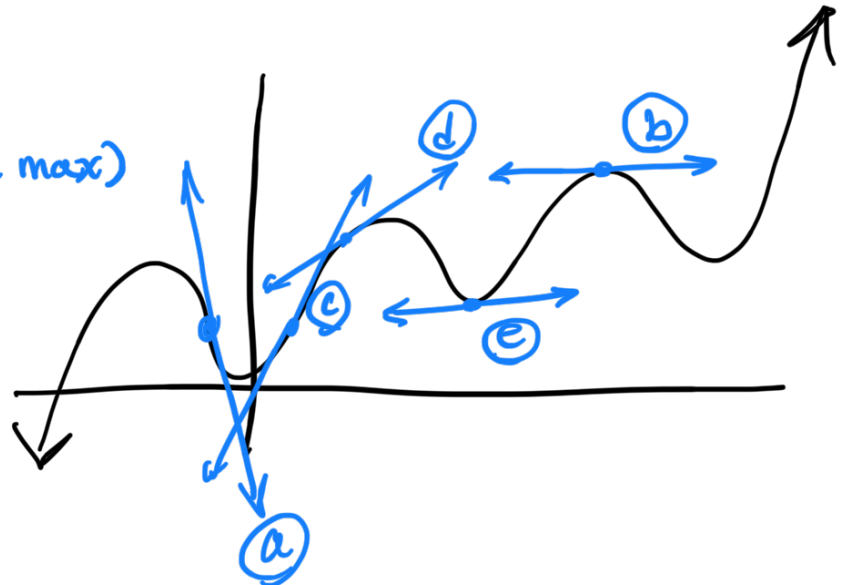
a.) negative slope (decreasing)

b.) slope of 0 at the top of a hill (local max)

c.) steeply positive slope

d.) slightly positive slope

e.) slope of 0 at the bottom of a hill (local min)



We will return to these ideas...

Keep in mind that horizontal lines have a slope of 0.