

Business Calculus
Class Notes
Section 1.8: Higher Order Derivatives

What is the rate of change of a function's rate of change?
What could that tell us?

Higher-Order Derivatives:

Since the derivative of a function is itself a function, we could find *its* derivative. In other words, we can find the derivative of the derivative, called the second derivative. And so on and so on. We can find the third derivative, fourth derivative, etc. for as long as they are defined.

As you see here, we will use prime notation to denote the first few derivatives. Take a look at the notation for higher derivatives.

DEFINITION Higher-Order Derivatives

Assuming $y = f(x)$ can be differentiated as often as necessary, the second derivative of f is

$$f''(x) = \frac{d}{dx}(f'(x)).$$

For integers $n \geq 1$, the n th derivative of f is

$$f^{(n)}(x) = \frac{d}{dx} f^{(n-1)}(x).$$

We answer the question, "How fast is the derivative function changing?"

Alternatively,
 $\frac{d^2 y}{dx^2}, \frac{d^3 y}{dx^3}, \text{etc}$

ex1: Find the first, second, third, and fourth derivatives for this function.

$$g(p) = \frac{5}{3}p^3 + 6\sqrt{p} = \frac{5}{3}p^3 + 6p^{1/2}$$

$$g'(p) = 5p^2 + 3p^{-1/2}$$

$$g''(p) = 10p - \frac{3}{2}p^{-3/2}$$

$$g'''(p) = 10 + \frac{9}{4}p^{-5/2}$$

$$g^{(4)}(p) = \frac{9}{4} \left(-\frac{5}{2}\right) p^{-7/2}$$

$$g^{(4)}(p) = -\frac{45}{8}p^{-7/2}$$

$$-\frac{3}{2} \left(-\frac{3}{2}\right) = \frac{9}{4}$$

A Common Application of Higher Derivatives: Velocity and Acceleration:

We have already seen how if $s(t)$ is the **position** function of an object in motion, then its derivative $s'(t)$ tells us the object's **velocity**, or rate of change of its position.

We have seen that the units for this rate of change are based on the fact that this is no more than slope (difference of $s(t)$ -values divided by difference of t -values). Hence if $s(t)$ is in feet and t is in seconds, then $s'(t)$ would be in feet per second. (velocity)

We will start to use the function name $v(t) = s'(t)$ to mean the object's **velocity**. What is the rate of change of this velocity function? In other words, what would we call $v'(t) = s''(t)$? What would the units be for such a function?

$$\frac{\text{ft/sec}}{\text{sec}} \text{ or } \text{ft/sec}^2$$

$\uparrow$ units $\uparrow$ units

We call $v'(t) = s''(t)$ the object's **acceleration** or $a(t)$.

Don't forget about the alternative notation

$$\frac{ds}{dt} \text{ and } \frac{d^2s}{dt^2}$$

expl 2: If we let a stone drop from a very high ledge, we can say that the distance it has fallen after t seconds is given by $s(t) = 5.3t^2$ for $0 \leq t \leq 10$.

a.) Find the distance it has fallen after 5 seconds.

$$s(5) = 5.3(5)^2 = 132.5 \text{ feet}$$

b.) Find its velocity and acceleration after t seconds.

$$v(t) = s'(t)$$

$$a(t) = v'(t)$$

$$v(t) = 5.3 \cdot 2t$$

$$a(t) = 10.6$$

$$v(t) = 10.6t$$

c.) Find its velocity and acceleration after 5 seconds.

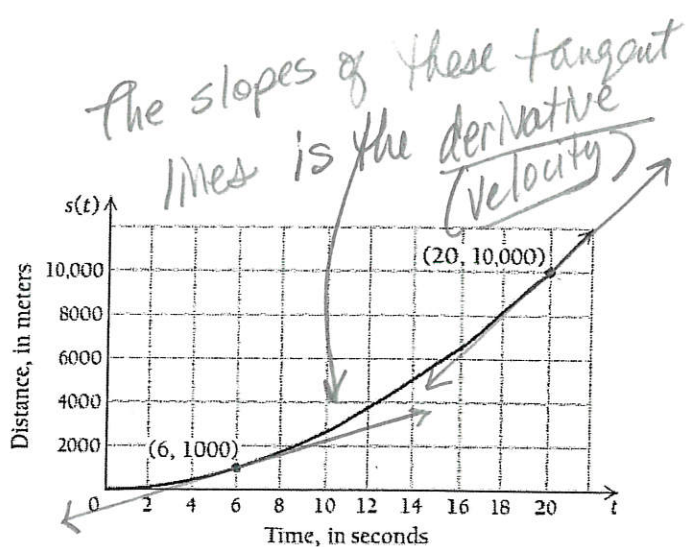
$$v(5) = 10.6(5) = 53 \text{ ft/sec}$$

$$a(5) = 10.6 \text{ ft/sec}^2$$

We will cover various functions and their rates of change (derivatives). A commonly explored one is $s(t)$, $v(t)$, and $a(t)$ we have just seen.

The book uses speed and velocity interchangeably. However, truly speed is the absolute value of velocity.

expl 3: Graphed to the right is an airplane's distance from its last point of rest. Draw in tangent lines at the points shown at $t = 6$ and $t = 20$.



- a.) Is the plane's velocity greater at $t = 6$ or $t = 20$? Explain.

$t = 20$ cause the slope is greater.

- b.) Is the plane's acceleration positive or negative? Explain.

The velocity over time is increasing — means acceleration is positive (rate of change in velocity)

Acceleration is the velocity function's rate of change.

expl 4: Nate's Necklaces noticed the number of necklaces sold t days after launching a new sales promotion is given by $N(t) = 3t^3 - 4t^2 + 2t$.

- a.) Find and interpret $N(3)$.

$$N(3) = 3 \cdot 3^3 - 4 \cdot 3^2 + 2 \cdot 3$$

$= 51$ — On day 3, we'll

Keep in mind that the units are necklaces (N) and days (t).

- b.) Find and interpret $N'(3)$.

$$N(t) = 3t^3 - 4t^2 + 2t$$

$$N'(t) = 9t^2 - 8t + 2$$

$$N'(3) = 9 \cdot 3^2 - 8 \cdot 3 + 2 = 59 \text{ necklaces/day}$$

The number of necklaces sold is changing at 59/day on day 3.

- c.) Find and interpret $N''(3)$.

$$N''(t) = 18t - 8$$

$$N''(3) = 18 \cdot 3 - 8$$

$$= 46 \text{ Necklaces/day/day}$$

The rate of change of number of necklaces is changing by 46 necklaces/day every day at day 3. ✓

