

The natural exponential function was jogging alone down a dark alley. Suddenly, a differential operator jumps from the shadows. Was our function worried? She was *not* as she knew he could do *nothing* to her!

Section 2.2: Derivatives of Natural Exponential Functions

Recall that the **natural exponential function** is $f(x) = e^x$.

Draw a quick graph and draw in a few tangent lines.

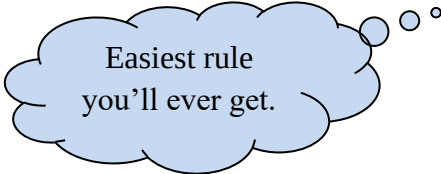
We are interested in the **rate of change** for this natural exponential function. Derivatives, ho!

The book does a good job of explaining this next gem.

Derivatives of Natural Exponential Functions:

A fun little tidbit about this **natural exponential function** is that it is its own derivative. Rather,

$$\frac{d}{dx}(e^x) = e^x$$



Easiest rule
you'll ever get.

But what if this exponent is *not* just x , but rather a function in x ? Then, we'd need the Chain Rule. So, we have this lovely nugget.

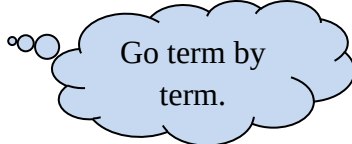
$$\frac{d}{dx}(e^{f(x)}) = e^{f(x)} \cdot f'(x)$$

Equivalently, if we think of this exponent as u which is a function in x , we would write this.

$$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$$

Let's do some examples.

expl 1: Find the derivative of $y = 4e^x + 3x^2$.



Go term by
term.

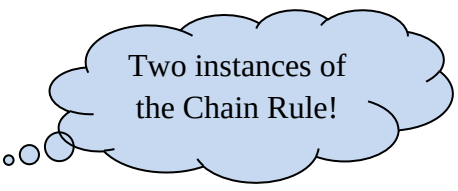
expl 2: Find the following.

a.) $\frac{d}{dx}(e^{3x^2+4x})$

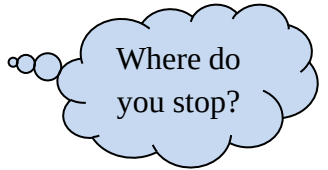
b.) $\frac{dy}{dt}$ for the function $f(t) = e^t + \sqrt{5e^{3t}}$

c.) $\frac{d}{dx}(5e^{3x} \cdot x^6)$

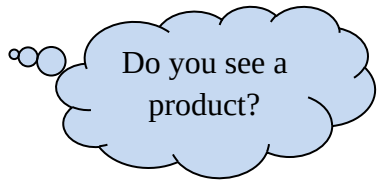
d.) $f''(x)$ for the function $f(x) = (x^4 + 8x^3 + 7)e^x$



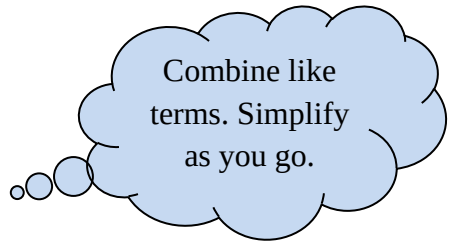
Two instances of
the Chain Rule!



Where do
you stop?



Do you see a
product?



Combine like
terms. Simplify
as you go.

Application: Compound Interest:

expl 3: Consider a compound interest account that compounds *continuously* with \$5,000 invested at 3.15% interest. Answer the following questions. Include units.

a.) What is the formula for the amount in the account after t years?

b.) What is the rate of change for the amount in the account after t years?

c.) What is the rate of change for this rate of change for the amount in the account after t years? Don't round.

d.) Evaluate both rates at 10 years. Round to two decimal places. (Do you see why?) Include units. Interpret.

e.) The original function has a point at (10, 6851.30). What does this mean? Find the equation of the tangent line to the curve at this point. Graph both on your calculator to check.