

Theorem 1

For a function $f(x)$ and L a real number,

$$\lim_{x \rightarrow a} f(x) = L \text{ if and only if } \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L.$$

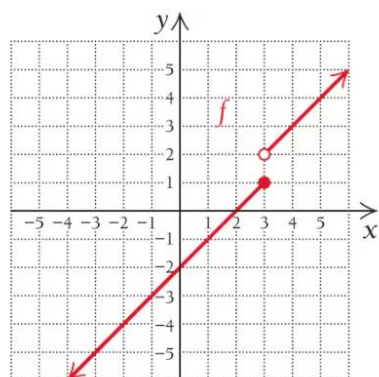
The $\lim_{x \rightarrow a} f(x)$ does not exist if the left-hand or right-hand limit does not exist or if the left-hand or right-hand limit exist but are different.

The existence of $\lim_{x \rightarrow a} f(x)$ does not require that $f(a)$ exists. If $f(a)$ does exist, it does not automatically follow that $\lim_{x \rightarrow a} f(x) = f(a)$. (See [Example 5](#).)

1.1 Examples done in class:

For Exercises 21 and 22, consider the function f given by

$$f(x) = \begin{cases} x - 2, & \text{for } x \leq 3, \\ x - 1, & \text{for } x > 3. \end{cases}$$



If a limit does not exist, state that fact.

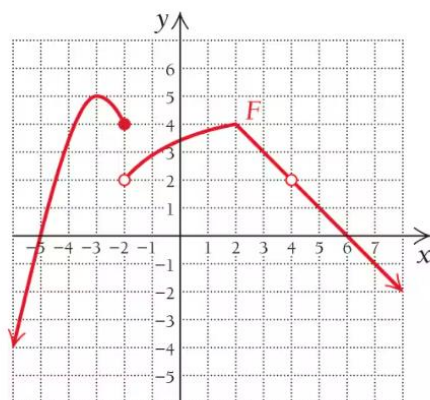
21. Find

a. $\lim_{x \rightarrow 3^-} f(x);$

b. $\lim_{x \rightarrow 3^+} f(x);$

c. $\lim_{x \rightarrow 3} f(x).$

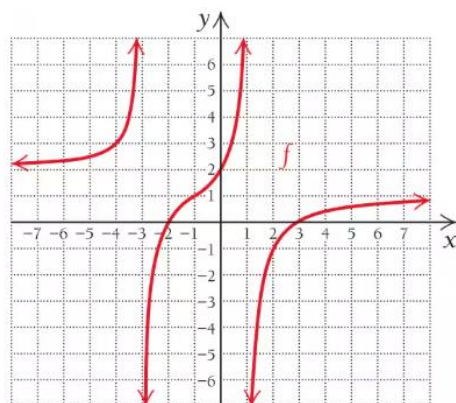
For Exercises 25–34, use the following graph of F to find each limit. When necessary, state that the limit does not exist.



29. $\lim_{x \rightarrow 4} F(x)$

30. $\lim_{x \rightarrow 6} F(x)$

For Exercises 57–66, use the following graph of f to find each limit. When necessary, state that the limit does not exist.



60. $\lim_{x \rightarrow -3} f(x)$