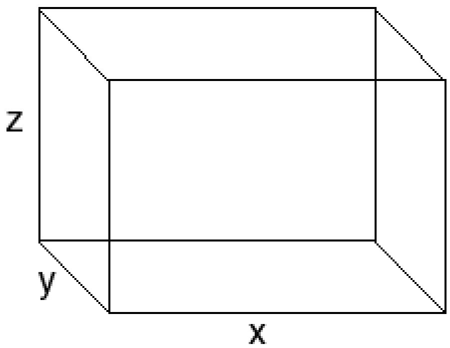
Lagrange multipliers NAMES:

Solve the following problem using Lagrange multipliers. The steps are outlined below.

The material for a rectangular box costs $2 per square foot for the top and $1 per square foot for the bottom and sides. Find the dimensions for which the volume of the box is 12 cubic feet and the cost of the materials is minimized. A picture is given below. The dimensions of the box are in feet.



1. The volume equation will be the constraint equation. Form an equation that shows the volume to be 12 cubic feet.

2. The equation for the cost of materials will be minimized. Notice the box has a top of xy square feet, two sides that are yz square feet, two sides that are xz square feet, and a bottom that is xy square feet. Using the fact that the top costs $2 per square foot and the sides and bottom cost $1 per square foot, form an equation that gives the cost of materials in terms of x, y, and z.

3. For question #1, did you get $xyz = 12$? This means that $g(x, y, z) = xyz$. Find the partial derivatives $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$, and $\frac{\partial g}{\partial z}$.

4. For question #2, did you get $f(x, y, z) = 3xy + 2yz + 2xz$? Find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$.

5a. Now form the equation $\frac{\partial f}{\partial x} = l \frac{\partial g}{\partial x}$.

5b. Now form the equation $\frac{\partial f}{\partial y} = l \frac{\partial g}{\partial y}$.

5c. Now form the equation $\frac{\partial f}{\partial z} = l \frac{\partial g}{\partial z}$.

6. Question 5 should have resulted in the equations $3y + 2z = lyz$, $3x + 2z = lxz$, and $2y + 2x = lxy$. We need to combine these equations so we can eliminate some variables. The following is a suggested path.

7. Solve the first two equations each for l z. Set these equal to each other and simplify.

8. Solve the last two equations (given in #6) each for l x. Set these equal to each other and simplify.

9. For question 7, you should have gotten $x = y$. For question 8, you should have gotten $\frac{3}{2}y = z$. Now we'll combine these two equations and $xyz = 12$.

Start with $xyz = 12$; substitute y for x and $\frac{3}{2}y$ for z. This gives you an equation entirely in y. Solve this to get $y = 2$. Use this to figure x and z. These values are the dimensions of the box that minimize the cost of the materials.