## Finite Math <br> Linear Programming Worksheet

We will work through these steps to solve the maximization problem in example 3 (page 221) as a class. Please do not work ahead.

This process relies on certain conditions. They are
a.) the objective function is to be maximized,
b.) all the variables are nonnegative (ie. $\mathrm{x}, \mathrm{y}$, and z are all greater than or equal to zero), and
c.) each linear constraint can be written so that the expression involving the variables is less than or equal to a nonnegative constant.

For example 3, the constraint inequalities are $2 \mathrm{x}+\mathrm{y}+2 \mathrm{z} \leq 14$

$$
2 x+4 y+z \leq 26
$$

$$
x+2 y+3 z \leq 28
$$

The objective function to be maximized is $\mathrm{P}=2 \mathrm{x}+2 \mathrm{y}+\mathrm{z}$.
Notice the above three conditions hold.

1. Rewrite the inequalities as equalities using slack variables $\mathrm{u}, \mathrm{v}$, and w .
2. Rewrite these equations so that all of the variables are on one side and the constant is alone on the other side of the equal sign. (This is actually already the case except for the objective function.)
3. Write these equations in a simplex tableau writing the coefficients of each variable in the appropriate column. The first wide row is used for the three constraint equations. The second row is used for the objective function

| $x$ | $y$ | $z$ | u | v | w | P | Constant |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

4. In the bottom row, find the most negative number. The corresponding column is called the pivot column. In our case there are two such columns, so we get to pick the one we'll use; we'll pick the x column.
5. Now divide each of the other entries in this column into the corresponding constant at the far right. Use the space below to record your calculations. The entry that gives you the smallest ratio is called the pivot element. Circle it in the tableau. We will essentially do Gauss-Jordan elimination using this entry as our pivot.
6. First, we need to make the pivot element into 1 . So divide the whole row by the pivot element. Rewrite the tableau, not changing the other rows. Circle the pivot element.

| x | y | z | u | v | w | P | Constant |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

7. Now we use Gauss-Jordan to make the rest of the entries in this column zeros. Rewrite the tableau. Be sure to record your row operations.

| x | y | z | u | v | w | P | Constant |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
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8. This completes one iteration of the process. We then check the bottom row as we did in step 4. If there are no negative numbers, we are done and we pick out our solution. This is not the case, so we go on.
9. Find the most negative number in the bottom row. The corresponding column is again called the pivot column. Now divide each of the other entries in this column into the corresponding constant at the far right. Use the space below to record your calculations. The entry that gives you the smallest ratio is again called the pivot element. Circle it in the tableau.
10. First, we need to make the pivot element into 1 . So divide the whole row by the pivot element. Rewrite the tableau, not changing the other rows. Circle the pivot element.

11. Now we use Gauss-Jordan to make the rest of the entries in this column zeros. Rewrite the tableau. Be sure to record your row operations.

| x | y | z | u | v | w | P | Constant |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |
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12. We again search for the most negative number in the bottom row. Notice this time there are no negative numbers. This indicates we are done. We can now pick out our solution. Recall we want values for $\mathrm{x}, \mathrm{y}$, and z (that maximize the objective function).
13. All the columns that are not composed entirely of zeros and ones correspond to nonbasic variables. In our case the nonbasic variables are $\mathrm{z}, \mathrm{u}$, and v . All the columns that are composed entirely of zeros and ones correspond to basic variables. In our case, the basic variables are $\mathrm{x}, \mathrm{y}, \mathrm{w}$, and P .
14. Write down the equations that correspond to the rows.
15. Assign the nonbasic variables ( $\mathrm{z}, \mathrm{u}$, and v ) the value of zero. Rewrite your equations with these variables gone. This allows us to solve for the basic variables.
16. This gives us $\mathrm{x}=5, \mathrm{y}=4, \mathrm{z}=0$, and $\mathrm{P}=18$. This tells us the ordered triplet $(5,4,0)$ is the point that maximizes the objective function P . The maximum value of P is 18 .
17. Notice that $w=15$. This gives us additional information about the constraint that involved w . It would have meaning if the problem had a context. The interpretation of values for the slack variables is discussed in example 4 in the textbook at the bottom of page 225.
